

# MODEL QUESTION PAPER MATHEMATICS HSSC-II (Based on Curriculum 2006) <br> SECTION - A (Marks 20) <br> Time allowed: $\mathbf{2 5}$ Minutes 

Note: Section-A is compulsory. All parts of this section are to be answered on the separately provid OMR Answer Sheet and should be completed in the first 20 minutes and handed over to $t$ Centre Superintendent. Do not use lead pencil.

Q1. Choose the correct answer by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1. What result occurs, in evaluating $\lim _{x \rightarrow 3} \frac{x^{3}-27}{x-3}$ ?
A) 9
B) -9
C) 27
D) does not exist
2. Which of the following represents an odd function?
A) $f(x)=\frac{3 x}{x^{2}+1}$
B) $f(x)=3 x^{4}-2 x^{2}+7$
C) $f(x)=\sin x+\cos x$
D) $f(x)=(x+2)^{2}$
3. Which of the following represents $f^{-1}(\sqrt{2})$, if $f(x)=\sqrt{2 \tan x}$ ?
A) $\frac{\pi}{4}$
B) $\frac{7 \pi}{20}$
C) $\frac{\pi}{2}$
D) $\frac{3 \pi}{4}$
4. If $f(x)=\cos x, \quad x \in\left(\frac{\pi}{2}, \pi\right)$ then what results $f^{\prime}\left(\frac{3 \pi}{4}\right)$ ?
A) $\frac{\sqrt{3}}{2}$
B) $\frac{1}{\sqrt{2}}$
C) $-\frac{\sqrt{3}}{2}$
D) $-\frac{1}{\sqrt{2}}$
5. In which of the following intervals, $f(x)=2 x^{2}-8 x+1$ increases its value?
A) $(-\infty, 2]$
B) $(-\infty, 0]$
C) $[0, \infty)$
D) $(2, \infty)$
6. For a function $f(x)=\sin (\sin x)$ what evaluates $f^{\prime}(0)$ ?
A) 1
B) 0
C) -1
D) does not exist
7. Which of the options represents $f^{\prime}(x)=e^{x}+\sin x+1$ and $f(0)=2$ ?
A) $f(x)=e^{x}+\cos x+x$
B) $f(x)=e^{x}-\cos x+x+2$
C) $f(x)=x e^{x-1}-\cos x+x+3$
D) $f(x)=e^{x}+\cos x$
8. What evaluates $\int_{0}^{\pi / 4} \frac{e^{\tan x}}{\cos ^{2} x} d x$ ?
A) $e-1$
B) $e$
C) $\frac{\pi}{4}$
D) 0
9. The graph of $f(x)=\int_{a}^{x} g(t) d t$ is shown in the figure. For what value of $x, f(x)$ has its maximum value?
A) $a$
B) $b$
C) $c$
D) $d$

10. Which of the following lines passes through $(-7,7),(-7,-7)$ and $(-7,0)$ ?
A) $x=-7$
B) $y=-7$
C) $x+y=-7$
D) $y=-x+7$
11. How many intercepts are there in the graph of $y=\frac{1}{x}$ ?
A) no intercepts
B) two $x$-intercepts
C) two $y$-intercepts
D) one $x$ and one $y$-intercept
12. At what angle lines $3 y=2 x+5$ and $3 x+2 y=8$ cut each other?
A) $\frac{\pi}{6}$
B) $\frac{\pi}{4}$
C) 0
D) $\frac{\pi}{2}$
13. Which of the following options does not satisfy $4 x-3 y<2$ ?
A) $(1,1)$
B) $(0,0)$
C) $(3,0)$
D) $(-2,1)$
14. What are the coordinates of center of a circle $x^{2}+y^{2}-8 x+12 y+21=0$ ?
A) $(4,6)$
B) $(-4,6)$
C) $(4,-6)$
D) $(-4,-6)$
15. What is the equation of axis of a parabola $y^{2}-2 y+8 x-23=0$ ?
A) $y=-1$
B) $x=3$
C) $y=1$
D) $x=-3$
16. If $(5,-2),(5,4)$ are the vertices of a hyperbola, then center of hyperbola is:
A) $(0,0)$
B) $(5,3)$
C) $(5,1)$
D) $(5,0)$
17. Which of the following represents a graph of $4 x^{2}+y^{2}-8 x+4 y-9=0$ ?
A) circle
B) ellipse
C) parabola
D) hyperbola
18. For what value of $\propto$, vectors $4 \underline{i}+3 \underline{j}-3 \underline{k}$ and $\propto \underline{i}+3 \underline{k}$ have the same magnitude?
A) $\pm 5$
B) 5
C) 25
D) -5
19. If vectors $3 \underline{i}-6 \underline{j}+\underline{k}$ and $2 \underline{i}-4 \underline{j}+\lambda \underline{k}$ are parallel to each other then, value of $\lambda$ is:
A) $\frac{2}{3}$
B) $\frac{3}{2}$
C) $-\frac{3}{2}$
D) $-\frac{2}{3}$
20. What is the projection of $\underline{i}-\underline{k}$ along $\underline{j}+\underline{k}$ ?
A) $\frac{1}{\sqrt{2}}$
B) $-\frac{1}{2}$
C) $-\frac{1}{\sqrt{2}}$
D) -1

Note: Sections ' $B$ ' and ' $C$ ' comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Write your answers neatly and legibly.

## SECTION - B (Marks 48)

Q2. Attempt all parts. Each part carries (04) marks.
i. If $f(x)=p x+q$ and $g(x)=\frac{1}{p}(x-q)$, then show that $f[g(x)]=g[f(x)]$

## OR

Given a function $f(x)= \begin{cases}7-4 x, & x<1 \\ x^{2}+2, & x \geq 1\end{cases}$
Evaluate $\operatorname{limit}_{x \rightarrow-6} f(x)$ and $\operatorname{limit}_{x \rightarrow 1} f(x)$ if exist.
ii. Let $f(x)=\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(3 x^{3}-2 x^{2}+x-1\right)$.

Use the rule for differentiating products and find $f^{\prime}(1)$.

## OR

Evaluate $\quad \lim _{\theta \rightarrow 0}\left[\frac{\sin \theta-\tan \theta}{\sin ^{3} \theta}\right]$
iii. Find $\frac{d y}{d x}$ if $y=\frac{(\sqrt{x}+1)\left(x^{\frac{3}{2}}-1\right)}{x^{\frac{1}{2}}-1}, x \neq 1$

OR
Find $\frac{d y}{d x}$ if $x=3+$ cost and $y=1-\sin t$
iv. In which interval a function $f(x)=\left(x^{2}-6 x+8\right)(x-5)$ increases and decreases?

## OR

Find area in the first quadrant bounded by $f(x)=4 x-x^{2}$ and the x -axis.
v. Use differentials to approximate the value of $(33)^{1 / 5}$

## OR

Evaluate $\int \frac{\ln x}{x^{2}} d x$
vi. A ball is moving in a straight line with acceleration $a=2 t+8$. Find velocity of the ball if $v=12 \mathrm{~m} / \mathrm{Sec}$ when $t=0$ and distance covered by the ball if $s=0$ when $t=0$.

## OR

Show that the points A $(0,0), B(2,1), C(3,3)$ and $D(1,2)$ are the vertices of a rhombus.
vii. A straight line passes through the point $(-4,8)$ and makes an angle $30^{\circ}$ with $x^{+}-$axis. Find equation of the straight line.

## OR

Find an angle between the pair of straight lines represented by a homogenous equation of second degree $6 x^{2}-5 x y-6 y^{2}=0$.
viii. Graph solution region of the following system of linear inequalities by shading.

$$
10 x+20 y \leq 140 ; \quad 6 x+18 y \geq 72 ; \quad x \geq 0 ; y \geq 0
$$

## OR

Graph the feasible region subject to the following constraints.
$6 x-8 y \leq 12 ; 3 x+4 y \geq 6 ; x \geq 0 ; y \geq 0$
ix. If lines $3 y=4 x-5$ and $3 y=-4 x-13$ are the diameters of a circle and a point $(-5,0)$ lies on the circle, then find equation of the circle.

## OR

Write an equation of a parabola having focus $(-2,1)$ and directrix $x=5$.
x. Find equations of tangent and normal to the ellipse $16 x^{2}+25 y^{2}=1$ at $\left(4, \frac{12}{5}\right)$.

## OR

Locate the vertices of an ellipse of eccentricity 0.8 whose foci lie at the points $(0, \pm 8)$.
Also find equation of the ellipse.
xi. Find the equation of hyperbola with center at origin, conjugate axis along $x$-axis, eccentricity $\sqrt{7}$ and sum of lengths of whose axes is 32 .

## OR

Volume of a parallelepiped determined by the vectors $\underline{u}=-2 \underline{i}+5 \underline{j}+3 \underline{k}$, $\underline{v}=\underline{i}+3 \underline{j}-2 \underline{k}$ and $\underline{w}=-3 \underline{i}+\underline{j}-2 \underline{k}$ is $78 \mathrm{~cm}^{3}$. Find height, if the base is taken as parallelogram determined by $\underline{u}$ and $\underline{v}$.
xii. Find angle between the vectors $\underline{u}=3 \underline{i}+\underline{j}-\underline{k}$ and $\underline{v}=2 \underline{i}-\underline{j}+\underline{k}$.

## OR

Find the work done by the force $F=\hat{\imath}-2 \hat{\jmath}+3 \hat{k}$ in moving an object along the line from the origin to the point $(2,-1,4)$.

## SECTION - C (Marks 32)

Note: Attempt all questions. Each question carries (08) marks.
Q3. Let $f(x)=\left\{\begin{array}{cll}4-x^{2} & \text { if } & x \leq 0 \\ 4+x & \text { if } & x>0\end{array}\right.$
Sketch the graph and justify the continuity/discontinuity of $f(x)$ at $x=0$.

## OR

Examine the function defined as $f(x)=\sin x+\cos ^{2} x$ for extreme values, where $x \in\left[0, \frac{\pi}{2}\right]$
Q4. Evaluate $\int \frac{x^{3}+4}{\left(x^{2}-1\right)\left(x^{2}+3 x+2\right)} d x$

## OR

Evaluate $\int_{1}^{3} \frac{\ln x \sin (\ln x)}{x} d x$
Q5. The diagram shows a line $l$ passing through points $A(-1,1), B(5,5)$ given that $C(4,1)$. Write equation of line $l$ in normal form and find area of triangle $A B C$.

## OR



A factory manufactures two types of cell phones, conventional and smartphone. Each cell phone requires the use of two operations assembly and finishing, and there are at most 24 hours available for each operation. A conventional phone requires 1 hour of assembly and 2 hours of finishing, while a smartphone needs 2 hours of assembly and 1 hour of finishing. Due to some restrictions, the company can make at the most 15 gadgets a day. If a profit of Rs. 1000 is realized for each conventional phone and Rs. 4000 for a smartphone, how many of each should be manufactured to maximize the profit?

Q6. Find the center, foci, eccentricity, vertices and equations of directrices of the conic $4 x^{2}-5 y^{2}+40 x-30 y-45=0$

## OR

Find angle between the following pair of curves.
$\frac{x^{2}}{6}-\frac{y^{2}}{10}=1$ and $\frac{x^{2}}{36}+\frac{y^{2}}{20}=1$

FBISE HSSC-II Examinations
Model Question Paper (Mathematics) new
(National Curriculum 2000)
Alignment of Questions with Student Learning Outcomes

| $\begin{gathered} \text { Sec-A } \\ \text { Q } 1 \\ \hline \end{gathered}$ | Contents and Scope | Student Learning Outcomes * | Cognitive Level ** | Allocated Marks |
| :---: | :---: | :---: | :---: | :---: |
| i | Limits of Important Functions. | (a) Limit of the following functions at $x=a$ $\frac{x^{n}-a^{n}}{x-a}, \quad \frac{x-a}{\sqrt{x}-\sqrt{a}}$ <br> and their application in evaluation of the limits of algebraic, exponential and trigonometric functions. | K | 1 |
| ii | Kinds of Functions. | To know the following types of functions: Algebraic, trigonometric, inverse trigonometric, hyperbolic, explicitly and implicitly defined functions, parametric representation of functions, even and odd functions. | U | 1 |
| iii | Composition and Inversion of Functions. | To know the meaning of the identity and constant functions and the techniques of composing the functions by algebraic methods. | U | 1 |
| iv | Differentiation of Functions other than Algebraic. | To find the derivatives of trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic functions using chain and other rules. | U | 1 |
| v | Extreme Values. | To find whether a function is increasing or decreasing in an interval. | U | 1 |
| vi | The Chain Rule. | Explanation and application of chain rule for composite functions and functions defined by parametric functions. | A | 1 |
| vii | Introduction to Integration. | To define integration as anti- derivative and to know simple standard integrals which directly follow from standard differentiation formulas and to apply them in the integration of functions of the same types. | A | 1 |
| viii | Definite Integrals. | To be able to differentiate between definite and indefinite integrals and to know and apply the theorems of definite integrals. | A | 1 |
| ix | Extreme Values. | To have the concept of maximum and minimum values and critical points of a function. | A | 1 |
| x | Equations of Straight lines. | To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines. | K | 1 |


| xi | Equations of Straight lines. | (d) Derivation of the following standard forms of the equations of the straight lines; slope intercept; point slope; two points; intercepts; normal and symmetric. | U | 1 |
| :---: | :---: | :---: | :---: | :---: |
| xii | Two or Three Straight lines. | Be able to find: <br> c) acute angle between two straight lines, condition of their parallelism and perpendicularity. | U | 1 |
| xiii | Linear Inequalities and their Graphs. | To know the meanings of linear inequalities in two variables and their solutions be graphically illustrated. | K | 1 |
| xiv | Circle | (a) To know the general form of the equation of circle $x^{2}+y^{2}+2 g x+2 f y+c=0$ and be able to find its center and radius. | K | 1 |
| xv | Parabola and its Elements. | To derive the standard forms of equations of parabolas and to draw their graphs and to find the elements. | U | 1 |
| xvi | Hyperbola and its Elements. | To know the concept of a hyperbola and its elements (center, foci, eccentricity, focal chord, latera recta, directrices, transverse and conjugate axes). | K | 1 |
| xvii | Ellipse and its Elements. | To know the concept of an ellipse and its elements (center, foci, eccentricity, vertices, major and minor axes, focal chord, latera recta, directrices). | U | 1 |
| xviii | Introduction of a Vector in Space. | To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ components of a vector $\underline{a}=a_{1} \hat{\imath}+a_{2} \hat{\jmath}+a_{3} \hat{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors. | U | 1 |
| xix | Introduction of a Vector in Space. | To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ components of a vector $\underline{a}=a_{1} \hat{\imath}+a_{2} \hat{\jmath}+a_{3} \hat{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors. | U | 1 |
| xx | Scalar Product of two Vectors. | (b) To know analytic expression of $\underline{a} \underline{b}$ i.e. $\underline{a}=a_{1} \hat{\imath}+a_{2} \hat{\jmath}+a_{3} \hat{k}$ and $\underline{b}=b_{1} \hat{\imath}+b_{2} \hat{\jmath}+b_{3} \hat{k}$ then $\underline{a} \cdot \underline{b}=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3}$ angle between two vectors; projection of one vector on another vector; properties of scalar product (parallel and perpendicular vectors). | K | 1 |
| $\begin{gathered} \text { Sec-B } \\ \text { Q } 2 \\ \hline \end{gathered}$ | Contents and Scope | Student Learning Outcomes* | Cognitive Level ** | Allocate d Marks |


| i | Composition and Inversion of <br> Functions. | To know the meaning of the identity and <br> constant functions and the techniques of <br> composing the functions by algebraic methods. <br> OR <br> To understand the concept of continuity of a <br> function at a point and in an interval intuitively, <br> explanation <br> of continuity and discontinuity through graphs. <br> Functions. | K+K Discontinuous |
| :--- | :--- | :--- | :--- | :--- |$\quad 4+4$


| vii | Equations of Straight lines. <br> OR <br> Homogeneous Equations of $2^{\text {nd }}$ degree in two variables $x$ and $y$. | (a) Concept of the slope of a line. <br> (d) Derivation of the following standard forms of the equations of the straight lines; slope intercept; point slope; two points; intercepts; normal and symmetric. <br> OR <br> To show that a $2^{\text {nd }}$ degree homogeneous equation in two variables $x$ and $y$ represents a pair of straight lines through the origin. To find the angle between these lines. | U+K | 4+4 |
| :---: | :---: | :---: | :---: | :---: |
| viii | Linear inequalities and their Graphs. | To determine graphically the region bounded by two or three simultaneous inequalities of nonnegative variables and shading the regions bounded by them. | U+U | 4+4 |
| ix | Circle <br> OR <br> Equation of a Parabola with given elements. | To find the equation of a circle in the form $(x-h)^{2}+(y-k)^{2}=r^{2}$ <br> OR <br> To find the equation of a parabola with the following given elements. <br> - focus and vertex <br> - focus and directrix <br> - vertex and directrix | U+U | 4+4 |
| x | Tangents and Normals to an ellipse. <br> OR Equation of an Ellipse with given elements. | (b) To find the equations of tangent and normal to an ellipse at a point. <br> OR <br> To find the equation of an ellipse with the following given elements. <br> - major and minor axes <br> - two points <br> - foci, vertices or lengths of a latera recta <br> - foci, minor axis or length of a latus rectum. | K+U | 4+4 |
| xi | Equation of a hyperbola with given elements. <br> OR <br> Scalar Triple Product of vectors. | - To find the equation of a hyperbola with the following elements: <br> - Transverse and conjugate axes with center at origin. <br> - Eccentricity, latera recta and transverse axis <br> - Focus, eccentricity and center <br> - Focus, center and directrix <br> - To convert equation of a hyperbola to the standard form by translation of axes and be able to find the elements. <br> OR <br> (d) To find the volume of a parallelepiped and regular tetrahedron. | U+U | 4+4 |


| xii | Scalar Product of two vectors. <br> OR <br> Scalar Product of two vectors. | To know analytic expression of $\bar{a} . \bar{b}$ i.e., <br> if $\begin{aligned} & \underline{a}=a_{1} \underline{i}+a_{2} \underline{j}+a_{3} \underline{k} \text { and } \\ & \underline{b}=b_{1} \underline{i}+b_{2} \underline{j}+b_{3} \underline{k} \text { then } \\ & \underline{a} \cdot \underline{b}=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3} \end{aligned}$ <br> $\overline{\text { angle between two vectors; vector; properties of }}$ scalar product (parallel and perpendicular vectors). <br> OR <br> Projection of one vector on another. | U+A | 4+4 |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Sec-C } \\ \text { Q No } \end{gathered}$ | Contents and Scope | Student Learning Outcomes* | Cogniti ve Level | Allocate <br> d <br> Marks |
| 3 | Continuous and Discontinuous Functions. <br> OR <br> Extreme Values. | To understand the concept of continuity of a function at a point and in an interval intuitively, explanation of continuity and discontinuity through graphs. <br> OR <br> - To have the concept of maximum and minimum values and critical points of a function. <br> - To know the second derivative test of maxima and minima. | U+U | 8+8 |
| 4 | Integration Involving Partial Fractions. <br> OR <br> Integration by substitution. | To be able to use partial fractions in integration of rational fractions having denominators consisting of: <br> (a) Linear factors <br> (b) Repeated linear factors (up to 3 ) <br> (c) Linear and non-repeated quadratic factors. OR <br> To know and be able to find the antiderivatives of functions by parts including the following forms $\sqrt{a^{2}-x^{2}}, \sqrt{a^{2}+x^{2}}, \sqrt{x^{2}-a^{2}}$ | K+K | 8+8 |
| 5 | Equations of Straight lines. <br> OR <br> Linear Programming. | (b) To find the slope of a line passing through two points. <br> (f) To transform the linear equation $a x+b y+$ $c=0$ in standard form. <br> (h) To find the area of a triangle whose vertices are given. <br> OR <br> To find the optimal solution of the linear objective functions by graphical methods. | A+A | 8+8 |


| 6 | Equation of Hyperbola with <br> given elements. | To convert equation of a hyperbola to the <br> standard form by translation of axes and to find <br> the elements. <br> OR |  |  |
| :---: | :--- | :--- | :---: | :---: |
|  | OR | Be able to know that two conics intersect in <br> Intersection of two conics. | 1) Four real points <br> 2) Two real points |  |
|  |  | 3) Two coincident real points <br> 4) One real point |  |  |

## * Student Learning Outcomes

National Curriculum for Mathematics Grades IX-XII, 2002

**Cognitive Level K: Knowledge<br>U : Understanding<br>A: Application

ASSESSMENT GRID FOR MATHEMATICS HSSC-II
Subject: Mathematics
Examination: Annual Class/Level: HSSC-II

| Topics |  |  |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{*} \\ & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{N}{O} \\ & \dot{\theta} \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Knowledge based | $\begin{aligned} & \hline 1 i(1) \\ & 2 i(4) \\ & 2 i(4) \\ & 2 i i(4) \\ & \hline \end{aligned}$ | 2iii (4) | $\begin{gathered} 2 v(4) \\ 4(8) \\ 4(8) \\ 2 v(4) \\ \hline \end{gathered}$ | $\begin{gathered} 1 x(1) \\ 2 v i i(4) \end{gathered}$ | 1xiii(1) | $\begin{gathered} 1 x i v(1) \\ 1 x v i(1) \\ 2 x(4) \end{gathered}$ | $1 \times x(1)$ | $\begin{gathered} 54 \\ (30 \%) \end{gathered}$ |
| Comprehension based | $\begin{gathered} 1 i i(1) \\ 1 i i i(1) \\ 3(8) \end{gathered}$ | $\begin{gathered} 1 i v(1) \\ 1 v(1) \\ 2 i i(4) \\ 2 i i i(4) \\ 2 i v(4) \\ 3(8) \end{gathered}$ |  | $\begin{gathered} 1 x i(1) \\ 1 x i i(1) \\ 2 v i(4) \\ 2 v i i(4) \end{gathered}$ | $\begin{aligned} & 2 v i i i(4) \\ & 2 v i i i(4) \end{aligned}$ | $\begin{gathered} \hline 1 x v(1) \\ 1 x v i i(1) \\ 2 i x(4) \\ 2 i x(4) \\ 2 x(4) \\ 2 x i(4) \\ 6(8) \\ 6(8) \\ \hline \end{gathered}$ | $\begin{gathered} 1 x v i i i(1) \\ 1 x i x(1) \\ 2 x i i(4) \end{gathered}$ | $\begin{gathered} 90 \\ (50 \%) \end{gathered}$ |
| Application based |  | $1 v i(1)$ | $\begin{gathered} \hline 1 v i i(1) \\ 1 v i i i(1) \\ 1 i x(1) \\ 2 i v(4) \\ 2 v i(4) \\ \hline \end{gathered}$ | 5(8) | 5(8) |  | $\begin{aligned} & 2 x i(4) \\ & 2 x i i(4) \end{aligned}$ | $\begin{gathered} 36 \\ (20 \%) \end{gathered}$ |
| Total marks for each topic | 23 | 27 | 35 | 23 | 17 | 40 | 15 | 180 |

$1,2,3$ etc. stands for question numbers
i, ii, iii etc. stands for part of question numbers
(1), (2), (3) etc. stands for marks of question papers

