Version No.			

ROLL NUMBER						

Answer Sheet No._____

Sign. of Candidate _____

Sign. of Invigilator _____

MODEL QUESTION PAPER

MATHEMATICS HSSC–I (Based on Curriculum 2006)

SECTION – A (Marks 20)
Time allowed: 25 Minutes

Note: Section-A is compulsory. All parts of this section are to be answered on the separate provided OMR Answer Sheet and should be completed in the first 20 minutes and hand over to the Centre Superintendent. Do not use lead pencil.

Q.1 Choose the correct answer by filling the relevant bubble for each question on the OMR Answer Sheet. Each part carries one mark.

(1) Complex number $\frac{1}{(2-i)^2}$, in the form $a+ib$ is:

A) $\frac{3}{25}+\frac{4}{25}i$ B) $\frac{3}{25}-\frac{4}{25}i$ C) $-\frac{4}{25}-\frac{3}{25}i$ D) $\frac{4}{25}-\frac{3}{25}i$

(2) What is the conjugate of $(1+i)^3$?

A) $-2+2i$ B) $-2-2i$ C) $2+2i$ D) $2-2i$

(3) For what value of k, $\begin{vmatrix} 2 & -1 & k \\ 3 & 1 & 2 \\ -1 & 3 & -2 \end{vmatrix} = 0$?

A) -2 B) 0 C) 1.2 D) 2

(4) What is the row rank of a matrix $\begin{bmatrix} 1 & 3 & 5 \\ 4 & 5 & 5 \\ 1 & 2 & 2 \end{bmatrix}$?

A) 0 B) 1 C) 2 D) 3

(5) For what value of h, vectors $\underline{a} = 3\underline{i} + \underline{j} - \underline{k}$ and $\underline{b} = h\underline{i} - 4\underline{j} + 4\underline{k}$ are parallel?

A) -12 B) 4 C) 8 D) 12

(6) What is the angle between two non-zero vectors $\underline{a}$ and $\underline{b}$, if $|\underline{a} \times \underline{b}| = 5$ and $\underline{a}.\underline{b} = 5\sqrt{2}$?

A) 30° B) 45° C) 60° D) 90°

(7) If $a_n = 5n+1$ then sum of n-terms of the series is:

A) $\frac{n}{2}$ B) $\frac{n}{2}(7+3n)$ C) $\frac{n}{2}(7+4n)$ D) $\frac{n}{2}(7+5n)$

(8) If the Harmonic Mean of 30 and y is 24, then value of y is:

A) 20 B) 30 C) 40 D) 50

(9) The sum of first three terms of a series $\sum_{r=6}^{100}(r-2)^2$ is:

A) 2 B) 5 C) 15 D) 77

(10) In how many ways, 5 friends can be seated at a round table?

A) 5! B) 4! C) C_2^5 D) P_1^5

(11) What will be the probability of losing a game if the winning probability is 0.3?

A) 0.5 B) 0.6 C) 0.7 D) 0.8

(12) Which of the following is a correct option for the validity of $(3-5x)^{-1/2}$?

A) $|x|<5$ B) $|x|<\frac{5}{3}$ C) $|5x|<1$ D) $|x|<\frac{3}{5}$

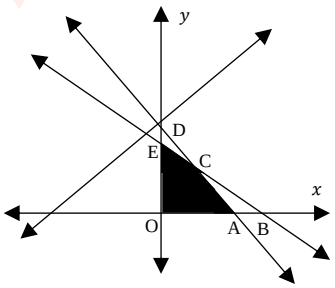
(13) If $f(x)=\frac{5}{x+3}$, then domain of $f^{-1}(x)$ is:

A) $\mathcal{R}$ B) $\mathcal{R}-\{0\}$ C) $\mathcal{R}-\{-3\}$ D) $\mathcal{R}-\{3\}$

(14) Which of the following are the corner points of the feasible region shown?

A) O,A,B,C,D,E B) O,A,C,E

C) A,C,E D) A,B,C,D,E



(15) If $\alpha+\beta+\gamma=180^\circ$ then $\operatorname{cosec}\alpha(\cos\beta\cos\gamma-\sin\beta\sin\gamma)$ is equal to:

A) $-\cot\alpha$ B) $\tan\alpha$ C) $\cot\alpha$ D) $\csc\alpha$

(16) Which of the following represents $2\cos 75^\circ\cos 15^\circ$?

A) $\frac{\sqrt{3}}{\sqrt{2}}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

(17) Which of the following represents $\left(\sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2}\right)$?

A) $\frac{\Delta}{a^3}$ B) $\frac{\Delta}{ac}$ C) $\frac{\Delta}{bc}$ D) $\frac{\Delta}{abc}$

(18) In triangle ABC (with usual notations) if $a=\sqrt{3},\ b=3$ and $\beta=60°$, then value of α is:

A) 30° B) 45° C) 60° D) 75°

(19) Period of $tan3\theta$ is same as that of:

A) $sec3\theta$ B) $\cot 6\theta$ C) $sin6\theta$ D) $tan9\theta$

(20) What is the range of a trigonometric function $y=-4+2sin(3x+5)$?

A) $[-2,-6]$ B) $[-4,2]$ C) $[-4,5]$ D)$[-6,-2]$

Federal Board HSSC-I Examination
MODEL QUESTION PAPER MATHEMATICS
(Based on Curriculum 2006)

Time allowed: 2:35 hours Total Marks Section B and C: 80

Note: Answer all parts from Section 'B' and all questions from Section 'C' on the **E-sheet**. Write your answers on the allotted/given spaces.

SECTION – B (Marks 48)

Q2. Attempt all parts. Each part carries **(04)** marks.

(i) Find the multiplicative inverse of a complex number $\left(\frac{4-i}{3+2i}\right)^{-2}$

OR

Sum the following series up to n-terms.

$1 + (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$

(ii) Use row operations to find the inverse of a square matrix $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 3 \\ 1 & -1 & 2 \end{bmatrix}$

OR

Using Properties of determinants, prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(iii) Find angle between two vectors $\underline{a} = 2\underline{i} - \underline{j} + 5\underline{k}$ and $\underline{b} = 3\underline{i} + \underline{j} - \underline{k}$.

OR

Find area of a triangle whose vertices are $(-1, 2, 3)$, $(1, -2, 3)$ and $(1, 2, -3)$.

(iv) Find two numbers whose harmonic mean is $\frac{24}{5}$ and geometric mean 6.

OR

Sum the series $1 \times 2^1 + 3 \times 2^2 + 5 \times 2^3 + 7 \times 2^4 + \ldots + 99 \times 2^{50}$

(v) If $5 \times P_3^n = 4 \times P_3^{n+1}$, find the value of n.

OR

In a factory, there are 100 units of a certain product, 5 of which are defective. If 3 units are selected from the 100 units at random, then what is the probability that none of them are defective?

(vi) Using Principle of Mathematical Induction, prove that $n^2 \geq 3n + 5$ for all positive integers $n \geq 5$.

OR

Find a constant term in the expansion of $\left(2\sqrt{x} + \frac{3}{4\sqrt{x}}\right)^{10}$

(vii) Find an equation of a parabola of the form $ax^2 + bx + c = 0$, which crosses $x - axis$ at $(-8, 0)$ and $(4, 0)$ and a point $(-2, -6)$ lies on it.

OR

For a real valued function f defined by $f(x) = \frac{3x-1}{x+1}, x \neq 1$. Find domain and range of $f^{-1}(x)$.

(viii) Graph the feasible region subject to the following constraints.

$6x - 8y \leq 12 \quad ; \quad 3x + 4y \geq 6 \quad ; \quad x \geq 0 \quad ; \quad y \geq 0$

OR

Find the point of intersection of the following functions graphically.

$f(x) = 2x + 1 \; ; \; g(x) = 2x^2 + 4x - 1$

(ix) Prove that $\cos 5\theta + 2\cos 3\theta + \cos\theta = 4\cos^2\theta \cos 3\theta$

OR

Find the domain, range and period of a trigonometric function $y = \frac{8}{3}\cot\left(\frac{2\pi}{5}x\right)$

(x) Find the interior angles of a triangle whose side measures are $5cm$, $6cm$ and $7cm$.

OR

Find radii R and r of circumscribed circles, respectively, of triangle ABC having side measures 8, 16 and 19.

(xi) Draw one cycle of the graph of $y = 3sin(\theta - 3\pi)$

OR

In triangle ABC (with usual notations),
prove that $\frac{s^2}{c}\left[\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}\right]\left[\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right] = (s - c)\cot\frac{\gamma}{2}$

(xii) Prove that $\cot^{-1}\left(\frac{1}{3}\right) - 2\tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{41}{3}\right)$

OR

Solve: $2\cos^4 x - 9\cos^2 x + 4 = 0$ where $x \in [0, 2\pi]$

SECTION – C (Marks 32)

Note: Attempt **ALL** questions. Each question carries (08) marks.

Q3. Solve the following simultaneous linear equations with complex coefficients.

$3x - (2 + i)y = i + 7 \quad ; \quad (2i - 1)x + (3i - 2)y = 2i + 1$

OR

Solve the following system of non-homogeneous linear equations using Gauss Jordan method.

$x + 5y + 3z = 7 \quad ; \quad 2x + 3y + z = 6 \quad ; \quad 3x - 2y + 2z = -3$

Q4. If 5, 7 and 9 are added to three consecutive terms of an A.P, the resulting numbers are in G.P. Find the numbers if their sum is 45.

OR

The sum of infinite number of terms in G.P is 17 and the sum of their squares is 51. Find the infinite Geometric series and its sum up to 5 terms.

Q5. If x is so small that its square and higher powers can be neglected, then show that

$$\frac{(1+x)^{\frac{3}{2}}(4-5x)^{\frac{1}{2}}}{(9+x)^{\frac{5}{2}}} \approx \frac{2}{243}\left(1 + \frac{43}{72}x\right)$$

OR

Find the maximum and minimum value of the function $f(x, y) = x + 3y$, subject to the following constraints

$2x + y \geq 4 \quad ; \quad 2x + 3y \leq 12 \quad ; \quad x + 2y \leq 16 \quad ; \quad x \geq 0 \quad ; \quad y \geq 0$

Q6. Prove that $24° + cos48° + cos96° + cos168° = \frac{1}{2}$.

OR

Solve graphically, the trigonometric equation: $sin(2x) = -x$, where $x \in [0, 2\pi]$

MATHEMATICS HSSC-I

Student Learning Outcomes

(National Curriculum 2006)

Sec A Q1	Contents and Scope	Student Learning Outcomes *	Cognitive Level **	Allocated Marks
i	1.1 Complex Numbers	(v) Define $\overline{z} = a - ib$ as the complex conjugate of $z = a + ib$.	K	1
ii	1.1 Complex Numbers	(iv) Carryout basic operations on complex numbers.	U	1
iii	2.3 Determinants	(iii) Define singular and non-singular matrices.	K	1
iv	2.5 Row and Column Operations	(v) Use row operations to find the inverse and the rank of a matrix.	U	1
v	3.1 Vectors in Plane	(iii) Give the following fundamental definitions using geometrical representation: • parallel vectors	K	1
vi	3.5 Dot or Scalar Product 3.6 Cross or Vector Product	$(viii)$ Use dot product to find the angle between two vectors. $(viii)$ Use cross product to find the angle between two vectors.	K	1
vii	4.4 Arithmetic Series	(ii) Establish the formula to find the sum to n terms of an arithmetic series.	U	1
$viii$	4.9 Harmonic Mean	(i) Define a harmonic mean.	K	1
ix	5.2 Arithmetico-Geometric Series	(i) Define arithmetico-geometric series	U	1
x	6.2 Permutation	(v) Find the arrangement of different objects around a circle.	A	1
xi	6.4 Probability	(ii) Recognize the formula for probability of occurrence of an event E, i.e. $P(E) = \frac{n(E)}{n(S)}, \quad 0 \leq P(E) \leq 1$	A	1
xii	7.3 Binomial Series	(ii) Expand $(1+x)^n$ in ascending powers of x and explain its validity/convergence for $\lvert x \rvert < 1$ where *n* is a rational number.	U	1
$xiii$	8.2 Inverse Function	Define inverse functions and demonstrate their domain and range with examples.	U	1
xiv	9.3 Feasible Region	(iii) Identify the feasible region of simple LP problems.	U	1
xv	10.2 Trigonometric Ratios of Allied Angles	(ii) Use fundamental law and its deductions to derive trigonometric ratios of allied angles.	U	1
xvi	10.4 Sum, Difference and Product of sine and cosine	(i) Express the product (of sines and cosines) as sums or differences (of sines and cosines).	U	1
$xvii$	10.3 Double, Half and Triple Angle Identities	Derive double angle, half angle and triple angle identities from fundamental law and its deductions.	K	1
$xviii$	11.1 Solving Triangles	(iii) Apply law of sines, law of cosines and law of tangents to solve oblique triangles.	A	1
xix	12.1 Period of Trigonometric Functions	(iii) Discuss the periodicity of trigonometric functions.	A	1

xx	12.1 Period of Trigonometric Functions	(iv) Find the maximum and minimum value of a given function of the type: • $a + bsin\theta$, • $a + bcos\theta$, • $a + bsin(c\theta + d)$, • $a + bcos(c\theta + d)$, • the reciprocals of above, where a, b, c and d are real numbers.	U	1
Sec B Q2	**Contents and Scope**	**Student Learning Outcomes ***	**Cognitive Level ****	**Allocated Marks**
i	1.3 Solution of equations **OR** 4.7 Geometric Series	(i) Solve the simultaneous linear equations with complex coefficients. **OR** ii) Find the sum of n terms of a geometric series.	U+U	4+4
ii	2.5 Row and Column operations **OR** 2.4 Properties of Determinants	(v) Use row operations to find the inverse and rank of a matrix. **OR** ii) Evaluate the determinant without expansion (i.e., using properties of determinants)	U+U	4+4
iii	3.5 Dot or Scalar Product 3.6 Cross or Vector Product **OR** 3.6 Cross or Vector Product	$(viii)$ Use dot product to find the angle between two vectors. $(viii)$ Use cross product to find the angle between two vectors. **OR** iv) Prove that the magnitude of A× B represents the area of a parallelogram with adjacent sides A and B.	K+K	4+4
iv	4.9 Harmonic Mean 4.6 Geometric Mean **OR** 5.2 Arithmetico-Geometric Series	(i) Define a harmonic mean. (i) Know geometric mean between two numbers. **OR** (ii) Find sum to n terms of the arithmetico-geometric series.	K+K	4+4
v	6.2 Permutation **OR** 6.4 Probability	(iv) Apply P_r^n to solve relevant problems of finding the number of arrangements of n objects taken r at a time (when all n objects are different and when some of them are alike). **OR** (ii) Recognize the formula for probability of occurrence of an event E, i.e. $P(E) = \frac{n(E)}{n(S)}, \quad 0 \leq P(E) \leq 1$	K+K	4+4
vi	7.1 Mathematical Induction **OR** 7.2 Binomial Theorem	(ii) Apply the principle to prove the statements, identities or formulae. **OR** (iv) Find the specified term in the expansion of $(x + y)^n$.	U+U	4+4
vii	8.3 Graphical Representation of Functions **OR** 8.2 Inverse Function	(v) Predict functions from their graphs (use the factor form to predict the equationof a function of the type $f(x) = ax^2 + bx + c$, if two points where the graph crosses $x - axis$ and third point on the curve, are given). **OR** Define inverse functions and demonstrate their domain and range with examples.	A+U	4+4
$viii$	9.3 Feasible Region **OR** 8.4 Intersecting Graphs	(ii) Define and show graphically the feasible region (or solution space) of an LP problem. **OR** i) Find the intersecting point graphically when intersection occurs between • a linear and a quadratic function.	U+U	4+4
ix	10.4 Sum, Difference and Product of sine and cosine	(ii) Express the sums or differences (of sines and cosines) as products (of sines and cosines).	U+K	4+4

	OR 12.1 Period of Trigonometric Functions	**OR** i) Find the domain and range of the trigonometric functions. iii) Discuss the periodicity of trigonometric functions.		
x	11.1 Solving Triangles **OR** 11.3 Circles Connected with Triangles.	(ii) Define an oblique triangle and prove • the law of cosines, • the law of sines, • the law of tangents, and deduce respective half angle formulae. (iii) Apply above laws to solve oblique triangles. **OR** (ii) Derive the formulae to find • circum-radius, • in-radius, and apply them to deduce different identities.	U+U	4+4
xi	12.2 Graphs of Trigonometric Functions **OR** 11.1 Solving Triangles	iv) Define periodic, even/odd and translation properties of the graphs of $sin\theta$, $cos\theta$ and $tan\theta$, i.e., $sin\theta$ has: • periodic property $sin(\theta \pm 2\pi) = sin\,\theta$, • odd property $sin(-\theta) = -sin\theta$, • translation property $\begin{cases} sin(\theta - \pi) = -sin\theta \\ sin(\pi - \theta) = sin\theta \end{cases}$ **OR** (ii) Define an oblique triangle and prove • the law of cosines, • the law of sines, • the law of tangents, and deduce respective half angle formulae. (iii) Apply above laws to solve oblique triangles.	A+K	4+4
xii	12.4 Inverse Trigonometric Functions **OR** 12.5 Solving General Trigonometric Equations	(v) Apply addition and subtraction formulae of inverse trigonometric functions to verify related identities. **OR** (i)Solve trigonometric equations and check their roots by substitution in the given trigonometric equations so as to discard extraneous roots.	A+A	4+4
Sec-C Q No.	**Contents and Scope**	**Student Learning Outcomes ***	**Cognitive Level ****	**Allocated Marks**
3	1.3 Solution of equations **OR** 2.6 Solving System of Linear Equations	i) Solve the simultaneous linear equations with complex coefficients. For example, $\begin{cases} 5z - (3+i)w = 7 - i \\ (2-i)z + ziw = -1 + i \end{cases}$ **OR** (iv) Solve a system of 3 by 3 non-homogeneous linear equations using Gauss-Jordan method (reduced echelon form).	K+U	8+8
4	4.2 Arithmetic Sequence 4.5 Geometric Sequence **OR**	(iii) Solve problems involving arithmetic sequence. (iii) Solve problems involving geometric sequence. **OR**	U+K	8+8
5	7.3 Binomial Series **OR** 9.4 Optimal Solution	(iii) Determine the approximate values of the binomial expansions having indices as –ve integers or fractions. **OR** (ii) Find optimal solution (graphical) through the following systematic procedure: Establish the mathematical formulation of LP problem, construct the graph, identify the feasible region, locate the solution points, evaluate the objective function, select the optimal solution and verify the optimal solution by actually substituting values of	U+U	8+8

		variables from the feasible region.		
6	10.4 Sum, Difference and Product of sine and cosine **OR** 12.3 Solving Trigonometric Equations Graphically	(ii) Express the sums or differences (of sines and cosines) as products (of sines andcosines). **OR** (ii)Solve graphically the trigonometric equations of the type • $sin\,\theta = \frac{\theta}{2}$ • $cos\,\theta = \theta$ • $\tan\theta = 2\theta\ when\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	A+A	8+8

* **Student Learning Outcomes**
National Curriculum for Mathematics Grades IX-XII, 2006
****Cognitive Level** K: Knowledge U: Understanding A: Application

Federal Board of Intermediate and Secondary Education

ASSESSMENT GRID FOR MODEL QUESTION PAPER

Level: HSSC-I Subject: Mathematics Curriculum: 2006 Examination: Annual 2024

Topics/Cognitive Level	1. Complex Numbers	2. Matrices and Determinants	3. Vectors	4. Sequences and Series	5. Miscellaneous Series	6. Permutation, Combination and Probability	7. Mathematical Inductions and Binomial Theorem	8. Functions and Graphs	9. Linear Programming	10. Trigonometric Identities of Sum and Difference of Angles	11. Application of Trigonometry	12. Graphs of Trigonometric And Inverse Trigonometric Functions and Solution of Trigonometric Equations	Total Marks for Each Assessment Objective
Knowledge based	$1i(1)$ $3(8)$	$1iii(1)$	$1v(1)$ $1vi(1)$ $2iii(4)$ $2iii(4)$	$1viii(1)$ $2iv(4)$ $4(8)$	$2iv(4)$	$2v(4)$ $2v(4)$				$1xvii(1)$	$2xi(4)$	$2ix(4)$	54 (30%)
Understanding based	$1ii(1)$ $2i(4)$	$1iv(1)$ $2ii(4)$ $2ii(4)$ $3(8)$		$1vii(1)$ $2i(4)$ $4(8)$	$1ix(1)$		$1xii(1)$ $2vi(4)$ $2vi(4)$ $5(8)$	$1xiii(1)$ $2vii(4)$ $2viii(4)$	$1xiv(1)$ $2viii(4)$ $5(8)$	$1xv(1)$ $1xvi(1)$ $2ix(4)$	$2x(4)$ $2x(4)$	$1xx(1)$	90 (50%)
Application based						$1x(1)$ $1xi(1)$		$2vii(4)$		$6(8)$	$1xviii(1)$	$1xix(1)$ $2xi(4)$ $2xii(4)$ $2xii(4)$ $6(8)$	36 (20%)
Total marks for each topic	**14**	**18**	**10**	**26**	**05**	**10**	**17**	**13**	**13**	**15**	**13**	**26**	**180**

- 1, 2, 3 etc. stands for question numbers
- i, ii, iii etc. stands for part of question numbers
- (1), (2), (3) etc. stands for marks of question papers