Federal Board HSSC-II Examination
Model Question Paper Mathematics
New Pattern (Curriculum 2000)

Version Number ☐☐☐☐

SECTION – A

Time allowed: 25 minutes   Marks: 20

Note: Section-A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Do not use lead pencil.

Q1. **Choose the correct answer by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.**

1. What result occurs, in evaluating $\lim\limits_{x \to 3} \frac{x^3 - 27}{x - 3}$?

    A) 9   B) -9 C) 27   D) does not exist

1. Which of the following represents an odd function?

    A) $f(x) = \frac{3x}{x^2 + 1}$ B) $f(x) = 3x^4 - 2x^2 + 7$

    C) $f(x) = sinx + cosx$ D) $f(x) = (x + 2)^2$

3. Which of the following represents $f^{-1}(\sqrt{2})$, if $f(x) = \sqrt{2}\tan x$?

    A) $\frac{\pi}{4}$ B) $\frac{7\pi}{20}$ C) $\frac{\pi}{2}$ D) $\frac{3\pi}{4}$

4. If $f(x) = \cos x, \ x \in \left(\frac{\pi}{2}, \pi\right)$ then what results $f'\left(\frac{3\pi}{4}\right)$?

    A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{\sqrt{2}}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{1}{\sqrt{2}}$

5. In which of the following intervals, $f(x) = 2x^2 - 8x + 1$ increases its value?

    A) $(-\infty, 2]$ B) $(-\infty, 0]$ C) $[0, \infty)$ D) $(2, \infty)$

6. For a function $f(x) = \sin(\sin x)$ what evaluates $f'(0)$?

    A) 1   B) 0   C) -1 D) does not exist

7. Which of the options represents $f'(x) = e^x + \sin x + 1$ and $f(0) = 2$?

    A) $f(x) = e^x + \cos x + x$ B) $f(x) = e^x - \cos x + x + 2$

    C) $f(x) = xe^{x-1} - \cos x + x + 3$ D) $f(x) = e^x + \cos x$

8. What evaluates $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx$?

    A) $e - 1$ B) e C) $\frac{\pi}{4}$ D) 0

9. The graph of $f(x) = \int_a^x g(t)dt$ is shown in the figure.

   For what value of x, $f(x)$ has its maximum value?

   A) a B) b C) c D) d
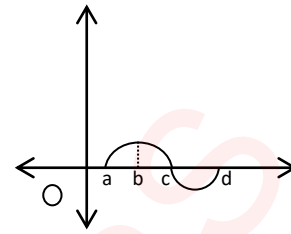
10. Which of the following lines passes through (–7, 7), (–7, –7) and (–7, 0)?

    A) $x = -7$ B) $y = -7$ C) $x + y = -7$ D) $y = -x + 7$

11. How many intercepts are there in the graph of $y = \frac{1}{x}$?

    A) no intercepts   B) two x–intercepts

    C) two y–intercepts   D) one x and one y–intercept

12. At what angle lines $3y = 2x + 5$ and $3x + 2y = 8$ cut each other?

    A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) 0 D) $\frac{\pi}{2}$

13. Which of the following options does not satisfy $4x - 3y < 2$?

    A) $(1, 1)$ B) $(0, 0)$ C) $(3, 0)$ D) $(-2, 1)$

14. What are the coordinates of center of a circle $x^2 + y^2 - 8x + 12y + 21 = 0$?

    A) $(4, 6)$ B) $(-4, 6)$ C) $(4, -6)$ D) $(-4, -6)$

15. What is the equation of axis of a parabola $y^2 - 2y + 8x - 23 = 0$?

    A) $y = -1$ B) $x = 3$ C) $y = 1$ D) $x = -3$

16. If (5, –2), (5, 4) are the vertices of a hyperbola, then center of hyperbola is:

    A) (0, 0)   B) (5, 3)   C) (5, 1)   D) (5, 0)

17. Which of the following represents a graph of $4x^2 + y^2 - 8x + 4y - 9 = 0$?

    A) circle   B) ellipse   C) parabola   D) hyperbola

18. For what value of $\propto$, vectors $4\underline{i} + 3\underline{j} - 3\underline{k}$ and $\propto \underline{i} + 3\underline{k}$ have the same magnitude?

    A) ± 5 B) 5   C) 25   D) –5

19. If vectors $3\underline{i} - 6\underline{j} + \underline{k}$ and $2\underline{i} - 4\underline{j} + \lambda\underline{k}$ are parallel to each other then, value of λ is:

    A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $-\frac{3}{2}$ D) $-\frac{2}{3}$

20. What is the projection of $\underline{i} - \underline{k}$ along $\underline{j} + \underline{k}$?

    A) $\frac{1}{\sqrt{2}}$ B) $-\frac{1}{2}$ C) $-\frac{1}{\sqrt{2}}$ D) -1

Time allowed: 2.35 hours   Total Marks: 80

Note: Sections 'B' and 'C' comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Write your answers neatly and legibly.

SECTION – B (Marks 48)

Q2.   Attempt all parts. Each part carries (04) marks.

i.   If $f(x) = px + q$ and $g(x) = \frac{1}{p}(x - q)$, then show that $f[g(x)] = g[f(x)]$.

ii.   Given a function $f(x) = \begin{cases} 7 - 4x, & x < 1 \\ x^2 + 2, & x \geq 1 \end{cases}$

Evaluate $\underset{x \to -6}{\text{limit}} f(x)$ and $\underset{x \to 1}{\text{limit}} f(x)$ if exist.

iii.   Let $f(x) = (x^4 - x^3 + x^2 - x + 1)(3x^3 - 2x^2 + x - 1)$.

Use the rule for differentiating products and find $f'(1)$.

OR

Find $\frac{dy}{dx}$ if $x = 3 + cost$ and $y = 1 - sint$

iv.   In which interval a function $f(x) = (x^2 - 6x + 8)(x - 5)$ increases and decreases?
OR

Find area in the first quadrant bounded by $f(x) = 4x - x^2$ and the x-axis.

v.   Use differentials to approximate the value of $(33)^{1/5}$

vi.   Evaluate $\int \frac{lnx}{x^2} dx$

vii.   A straight line passes through the point $(- 4, 8)$ and makes an angle $30°$ with

$x^+ - axis$. Find equation of the straight line.

OR

Find an angle between the pair of straight lines represented by a homogenous equation of second degree $6x^2 - 5xy - 6y^2 = 0$.

viii. Graph solution region of the following system of linear inequalities by shading.

$10x + 20y \leq 140$; $6x + 18y \geq 72$; $x \geq 0$; $y \geq 0$

ix.   If lines $3y = 4x - 5$ and $3y = -4x - 13$ are the diameters of a circle and a point $(-5, 0)$ lies on the circle, then find equation of the circle.

x.   Write an equation of a parabola having focus $(- 2, 1)$ and directrix $x = 5$.

OR

Find equations of tangent and normal to the ellipse $16x^2 + 25y^2 = 1$ at $\left(4, \frac{12}{5}\right)$.

xi. Find the equation of hyperbola with center at origin, conjugate axis along x-axis, eccentricity $\sqrt{7}$ and sum of lengths of whose axes is 32.

xii. Volume of a parallelepiped determined by the vectors $\underline{u} = -2\underline{i} + 5\underline{j} + 3\underline{k}$, $\underline{v} = \underline{i} + 3\underline{j} - 2\underline{k}$ and $\underline{w} = -3\underline{i} + \underline{j} - 2\underline{k}$ is $78\ cm^3$. Find height, if the base is taken as parallelogram determined by $\underline{u}$ and $\underline{v}$.

OR

Find angle between the vectors $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$.

SECTION – C (Marks 32)

Note: Attempt all questions. Each question carries (08) marks.

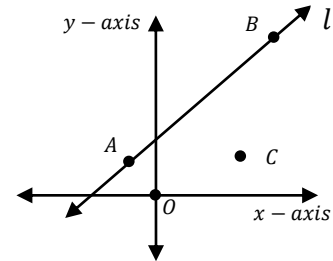Q3. Let $f(x) = \begin{cases} 4 - x^2 & if \quad x \le 0 \\ 4 + x & if \quad x > 0 \end{cases}$

Sketch the graph and justify the continuity/discontinuity of $f(x)$ at $x = 0$.

OR

Examine the function defined as $f(x) = \sin x + \cos^2 x$ for extreme values, where $x \in \left[0, \frac{\pi}{2}\right]$

Q4. Evaluate $\int \frac{x^3 + 4}{(x^2 - 1)(x^2 + 3x + 2)}\, dx$

Q5. The diagram shows a line l passing through points $A(-1, 1), B(5, 5)$ given that $C(4, 1)$. Write equation of line l in normal form and find area of triangle ABC.

OR

A factory manufactures two types of cell phones, conventional and smartphone. Each cell phone requires the use of two operations assembly and finishing, and there are at most 24 hours available for each operation. A conventional phone requires 1 hour of assembly and 2 hours of finishing, while a smartphone needs 2 hours of assembly and 1 hour of finishing. Due to some restrictions, the company can make at the most 15 gadgets a day. If a profit of Rs. 1000 is realized for each conventional phone and Rs. 4000 for a smartphone, how many of each should be manufactured to maximize the profit?

Q6. Find the center, foci, eccentricity, vertices and equations of directrices of the conic

$4x^2 - 5y^2 + 40x - 30y - 45 = 0$

FBISE HSSC-II Examinations
Model Question Paper (Mathematics)new
(National Curriculum 2000)
Alignment of Questions with Student Learning Outcomes

Sec-A	Q 1	Contents and Scope	Student Learning Outcomes *	Cognitive Level **	Allocated Marks
	i	Limits of Important Functions	(a) Limit of the following functions at $x = a$ $$\frac{x^n - a^n}{x - a}, \qquad \frac{x - a}{\sqrt{x} - \sqrt{a}}$$ and their application in evaluation of the limits of algebraic, exponential and trigonometric functions.	K	1
	ii	Kinds of Functions	To know the following types of functions: Algebraic, trigonometric, inverse trigonometric, hyperbolic, explicitly and implicitly defined functions, parametric representation of functions, even and odd functions.	U	1
	iii	Composition and Inversion of Functions	To know the meaning of the identity and constant functions and the techniques of composing the functions by algebraic methods.	U	1
	iv	Differentiation of Functions other than Algebraic	To find the derivatives of trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic functions using chain and other rules.	U	1
	v	Extreme Values	To find whether a function is increasing or decreasing in an interval.	U	1
	vi	The Chain Rule	Explanation and application of chain rule for composite functions and functions defined by parametric functions.	A	1
	vii	Introduction to Integration	To define integration as anti-derivative and to know simple standard integrals which directly follow from standard differentiation formulas and to apply them in the integration of functions of the same types.	A	1
	viii	Definite Integrals	To be able to differentiate between definite and indefinite integrals and to know and apply the theorems of definite integrals.	A	1
	ix	Extreme Values	To have the concept of maximum and minimum values and critical points of a function.	A	1

		x	Equations of Straight lines	To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines.	K	1
		xi	Equations of Straight lines	(d) Derivation of the following standard forms of the equations of the straight lines; slope intercept; point slope; two points; intercepts; normal and symmetric.	U	1
		xii	Two or Three Straight lines	Be able to find: c) acute angle between two straight lines, condition of their parallelism and perpendicularity.	U	1
		xiii	Linear Inequalities and their Graphs	To know the meanings of linear inequalities in two variables and their solutions be graphically illustrated.	K	1
		xiv	Circle	(a) To know the general form of the equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and be able to find its center and radius.	K	1
		xv	Parabola and its Elements	To derive the standard forms of equations of parabolas and to draw their graphs and to find the elements.	U	1
		xvi	Hyperbola and its Elements	To know the concept of a hyperbola and its elements (center, foci, eccentricity, focal chord, latera recta, directrices, transverse and conjugate axes).	K	1
		xvii	Ellipse and its Elements	To know the concept of an ellipse and its elements (center, foci, eccentricity, vertices, major and minor axes, focal chord, latera recta, directrices).	U	1
		xviii	Introduction of a Vector in Space	To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ components of a vector $\underline{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors.	U	1
		xix	Introduction of a Vector in Space	To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ components of a vector $\underline{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors.	U	1

				Cognitive Level **	Allocated Marks
	xx	Scalar Product of two Vectors	(b) To know analytic expression of $\underline{a}.\underline{b}$ i.e. $\underline{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\underline{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ then $\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ angle between two vectors; projection of onevector on another vector; properties of scalarproduct (parallel and perpendicular vectors)	K	1
Sec-B	**Q 2**	**Contents and Scope**	**Student Learning Outcomes ***	**Cognitive Level ****	**Allocated Marks**
	i	Composition and Inversion of Functions	To know the meaning of the identity and constant functions and the techniques of composing the functions by algebraic methods.	K	4
	ii	Continuous and Discontinuous Functions	To understand the concept of continuity of a function at a point and in an interval intuitively,explanation of continuity and discontinuity through graphs.	U	4
	iii	Theorems on Differentiation	To establish the theorems on differentiation sum, difference, product and quotient of functions and their application, differentials of $y = (ax + b)$ where n is a negative integer, using quotient theorem.	U	4
	iii	The Chain Rule	Explanation and application of chain rule for composite functions and functions defined by parametric functions.	U	4
	iv	Extreme values	To find whether a function is increasing or decreasing at a point and in an interval.	A	4
	iv	Application of Definite Integrals	To be able to calculate areas bounded by the curve and $x - axis$.	A	
	v	Differentials	Simple application of differentials in finding the approximate values of irrational numbers and $sinx, cosx$, when $x = 29°, 46°, 62°,$ etc.	A	4
	vi	Integration by Parts	To know and be able to find the anti-derivatives of functions by parts including the standard forms.	K	4

	vii	Equations of Straight lines	(a) Concept of the slope of a line. (d) Derivation of the following standard forms of the equations of the straight lines; slope intercept; point slope; two points; intercepts; normal and symmetric.	K	4
	vii	Homogeneous Equations of 2nd degree in two variables x and y	To show that a 2nd degree homogeneous equation in two variables x and y represents a pair of straight lines through the origin. To find the angle between these lines.	K	4
	viii	Linear inequalities and their Graphs	To determine graphically the region bounded by two or three simultaneous inequalities of non-negative variables and shading the regions bounded by them.	U	4
	ix	Circle	To find the equation of a circle in the form $(x - h)^2 + (y - k)^2 = r^2$	U	4
	x	Equation of a Parabola with given elements	To find the equation of a parabola with the following given elements. • focus and vertex • focus and directrix • vertex and directrix	K	4
	x	Tangents and Normals to an ellipse	(c) To find the equations of tangent and normal to an ellipse at a point.	K	4
	xi	Equation of a hyperbola with given elements	• To find the equation of a hyperbola with the following elements: ▪ Transverse and conjugate axes with center at origin. ▪ Eccentricity, latera recta and transverse axis ▪ Focus, eccentricity and center ▪ Focus, center and directrix • To convert equation of a hyperbola to the standard form by translation of axes and be able to find the elements.	U	4
	xii	Scalar Triple Product of vectors	(d) To find the volume of a parallelepiped and regular tetrahedron.	U	4
	xii	Scalar Product of two vectors	To know analytic expression of $\bar{a}.\bar{b}$ i.e. if $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ then $\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ angle between two vectors; projection of one vector on another vector; properties of scalar product (parallel and perpendicular vectors)	U	4

Sec-C	Q No	Contents and Scope	Student Learning Outcomes *	Cognitive Level **	Allocated Marks
	3	Continuous and Discontinuous Functions	To understand the concept of continuity of a function at a point and in an interval intuitively, explanation of continuity and discontinuity through graphs.	U	8
	3	Extreme Values	• To have the concept of maximum and minimum values and critical points of a function. • To know the second derivative test of maxima and minima.	U	8
	4	Integration Involving Partial Fractions	To be able to use partial fractions in integration of rational fractions having denominators consisting of: (a) Linear factors (b) Repeated linear factors (up to 3) (c) Linear and non-repeated quadratic factors	K	8
	5	Equations of Straight lines	(b) To find the slope of a line passing through two points. (f) To transform the linear equation $ax + by + c = 0$ in standard form. (h) To find the area of a triangle whose vertices are given.	A	8
	5	Linear Programming	To find the optimal solution of the linear objective functions by graphical methods.	A	8
	6	Equation of Hyperbola with given elements	To convert equation of a hyperbola to the standard form by translation of axes and to find the elements.	U	8

* Student Learning Outcomes

National Curriculum for Mathematics Grades IX-XII, 2002

**Cognitive Level K: Knowledge   U: Understanding   A: Application

ASSESSMENT GRID FOR MATHEMATICS HSSC-II

Subject: Mathematics   **Examination: Annual**   **Class/Level: HSSC-II**   **Year: 2022   Code:**

Topics	1. Functions and Limits	2. Differentiation	3. Integration	4. Introduction to Analytic Geometry	5. Linear Inequalities and Linear Programming	6. Conic Section	7. Vectors	Total marks of each assessment objectives
Assessment Objective								
Knowledge based	1i(1) 2i(4)		2vi(4) 4(8)	1x(1) 2vii(4) 2vii(4)	1xiii(1)	1xiv(1) 1xvi(1) 2x(4) 2x(4)	1xx(1)	38 (28%)
Comprehension based	1ii(1) 1iii(1) 2ii(4) 3(8)	1iv(1) 1v(1) 2iii(4) 2iii(4) 3(8)		1xi(1) 1xii(1)	2viii(4)	1xv(1) 1xvii(1) 2ix(4) 2xi(4) 6(8)	1xviii(1) 1xix(1) 2xii(4) 2xii(4)	66 (49%)
Application based		1vi(1) 2iv(4)	1vii(1) 1viii(1) 1ix(1) 2iv(4) 2v(4)	5(8)	5(8)			32 (23%)
Total marks for each topic	19	23	23	19	13	28	11	136

➢   1, 2, 3 etc. stands for question numbers
➢   i, ii, iii etc. stands for part of question numbers

➢ (1), (2), (3) etc. stands for marks of question papers