

Version No.			

ROLL NUMBER						

- 0  0  0  0  
 1  1  1  1  
 2  2  2  2  
 3  3  3  3  
 4  4  4  4  
 5  5  5  5  
 6  6  6  6  
 7  7  7  7  
 8  8  8  8  
 9  9  9  9

- 0  0  0  0  0  0  
 1  1  1  1  1  1  
 2  2  2  2  2  2  
 3  3  3  3  3  3  
 4  4  4  4  4  4  
 5  5  5  5  5  5  
 6  6  6  6  6  6  
 7  7  7  7  7  7  
 8  8  8  8  8  8  
 9  9  9  9  9  9

Answer Sheet No. \_\_\_\_\_

Sign. of Candidate \_\_\_\_\_

Sign. of Invigilator \_\_\_\_\_

### MATHEMATICS HSSC-I (3<sup>rd</sup> Set)

#### SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

**Q.1 Fill the relevant bubble for each part. All parts carry one mark.**

- (1) What is simplest form of  $[\sqrt{2}(1-i)]^4$  ?  
 A. 16  B. -16   
 C. 8  D. -8
- (2) Logically  $\sim (p \leftrightarrow q)$  is equivalent to:  
 A.  $q \leftrightarrow p$   B.  $p \leftrightarrow \sim q$    
 C.  $\sim q \leftrightarrow \sim p$   D.  $\sim p \leftrightarrow \sim q$
- (3) Under which of the following operations, the set  $A = \{-1, 0, 1\}$  is closed?  
 A. Addition  B. Subtraction   
 C. Multiplication  D. Division
- (4) Rank of matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$  is:  
 A. 1  B. 3   
 C. 2  D. 4
- (5) Which of the following represents the root of equation  $x^4 - 1 = 0$  ?  
 A.  $1, -1, w, w^2$   B.  $1, -1, i, -i$    
 C.  $0, 1, -w, -w^2$   D.  $2, -2, 2i, -2i$
- (6) What is the solution of  $2^{2x} - 3 \times 2^{x+2} + 32 = 0$  ?  
 A.  $\{2, 3\}$   B.  $\{-2, 3\}$    
 C.  $\{2, -3\}$   D.  $\{-2, -3\}$
- (7) The partial fraction of  $\frac{6+7x}{x^3+8}$  will be in the form:  
 A.  $\frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4}$   B.  $\frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$    
 C.  $\frac{A}{x+2} + \frac{Bx+C}{x^2+2x+4}$   D.  $\frac{A}{x+2} + \frac{Bx+C}{x^2+4}$

- (8) For what value of  $k$ , if  $\frac{1}{2}, \frac{1}{k-1}, \frac{1}{6}$  are in A.P
- A. 2  B. 3   
 C. 4  D. 5
- (9) What will be the value of  $\sum_{k=1}^n k^3$ ?
- A.  $\frac{n(n+1)}{2}$   B.  $\frac{n(n+1)(2n+1)}{6}$    
 C.  $\frac{n^2(n+1)^2}{4}$   D.  $\frac{n^2(n-1)}{2}$
- (10) What will be the probability of getting a number between 1 and 50 that is divisible by 1 and the number itself?
- A. 0.3  B. 0.35   
 C. 0.40  D. 0.32
- (11) For any natural number  $n$ ,  $7^n - 3^n$  is divisible by:
- A. 3  B. 4   
 C. 5  D. 7
- (12) In the expansion of  $(1+x)^{60}$ , the sum of coefficient of odd power of  $x$  is:
- A.  $2^{60}$   B.  $2^{59}$    
 C.  $2^{61}$   D.  $2^{120}$
- (13) Which one of the following is the simplified form of  $(1 - \sin x) \left(1 + \frac{1}{\sin x}\right)$ ?
- A.  $\sin x \cos x$   B.  $\sin x \tan x$    
 C.  $\cos x \cot x$   D.  $\csc x \cot x$
- (14) Which are Co-terminal angles?
- A.  $\frac{\pi}{3}, \frac{4\pi}{3}$   B.  $\frac{\pi}{3}, \frac{5\pi}{3}$    
 C.  $\frac{\pi}{3}, \frac{13\pi}{3}$   D.  $\frac{\pi}{3}, \frac{17\pi}{3}$
- (15) If  $\cos 47^\circ = z$ , express  $\cos 94^\circ$  in term of  $z$
- A.  $2z\sqrt{1-z^2}$   B.  $2z^2 - 1$    
 C.  $2z^2 + 1$   D.  $2z\sqrt{z^2 - 1}$
- (16) What is the domain of  $y = 3\tan 4x$ ?
- A.  $R - \left\{(2n+1)\frac{\pi}{2}; n \in Z\right\}$   B.  $R - \left\{(2n+1)\frac{\pi}{6}; n \in Z\right\}$    
 C.  $R - \left\{(2n+1)\frac{\pi}{8}; n \in Z\right\}$   D.  $R$
- (17) If area of  $\Delta ABC = 41.6$ ,  $a = 6$ ,  $b = 18$  then  $\angle C$  is:
- A.  $50^\circ 23'$   B.  $23^\circ 39'$    
 C.  $39^\circ 36'$   D.  $37^\circ 36'$
- (18) What is the angle of elevation of the sun when 10m high pole cast a shadow of 8m?
- A.  $52^\circ 7'$   B.  $51^\circ 20'$    
 C.  $53^\circ 20'$   D.  $54^\circ 6'$
- (19) The value of  $\cos \left[ \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) - \frac{\pi}{2} \right]$  is:
- A. 0  B.  $\frac{1}{\sqrt{2}}$    
 C.  $-\frac{1}{\sqrt{2}}$   D. 1
- (20) Solution set of  $\sec x = -\frac{2}{\sqrt{3}}$  is:
- A.  $\left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\}$   B.  $\left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}$    
 C.  $\left\{ \frac{5\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{3} + 2n\pi \right\}$   D.  $\left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\}$

**Model Question Paper (3<sup>rd</sup> Set Solution)**  
**Mathematics HSSC-I**

**Section – A**

1	B	2	B	3	C	4	B	5	B	6	A	7	A	8	C	9	C	10	A
11	B	12	B	13	D	14	B	15	B	16	C	17	A	18	B	19	B	20	D

**SECTION-B**

**Q2.**

(i)  $z = \frac{5-3i}{1+i} \times \frac{1-i}{1+i} = \frac{5-8i+3}{1-(-1)} = 1 - 4i$

a) Modulus of  $z = |z| = \sqrt{(1)^2 + (-4)^2} = \sqrt{17}$

b) Argument of  $z = \tan\theta = -\frac{4}{1} \Rightarrow \theta = 275.60^\circ$

(ii) **Closure Property:**

Let  $x, y$  be any two elements of  $G$  such that  $x = a + \sqrt{5}b, y = c + \sqrt{5}d \quad \forall a, b, c, d \in Q$

$$x + y = (a + \sqrt{5}b) + (c + \sqrt{5}d) = (a + c) + \sqrt{5}(b + d)$$

Since  $a + c$  and  $b + d$  are element of  $Q$

$$\Rightarrow [(a + c) + \sqrt{5}(b + d)] \in G$$

$\therefore G$  is closed with respect to  $' + '$ .

**Associative Property:**

Let  $(a + \sqrt{5}b), (c + \sqrt{5}d), (e + \sqrt{5}f) \in G, \quad \forall a, b, c, d, e, f \in Q$

$$\text{L.H.S} = [(a + \sqrt{5}b) + (c + \sqrt{5}d)] + (e + \sqrt{5}f)$$

$$= [(a + c) + \sqrt{5}(b + d)] + (e + \sqrt{5}f)$$

$$= (a + c + e) + \sqrt{5}(b + d + f) \quad \rightarrow \text{eqn(1)}$$

$$\text{R.H.S} = (a + \sqrt{5}b) + [(c + \sqrt{5}d) + (e + \sqrt{5}f)]$$

$$= (a + \sqrt{5}b) + [(c + e) + \sqrt{5}(d + f)]$$

$$= (a + c + e) + \sqrt{5}(b + d + f) \quad \rightarrow \text{eqn(2)}$$

From Equations (1) & (2),  $G$  is Associative with respect to  $' + '$ .

**Existence of Identity:**

$\forall (a + \sqrt{5}b) \in G$ , there exist  $(0 + \sqrt{5}0) \in G$  such that

$$[(a + \sqrt{5}b) + (0 + \sqrt{5}0)] = [(0 + \sqrt{5}0) + (a + \sqrt{5}b)] = (a + \sqrt{5}b)$$

$\therefore (0 + \sqrt{5}0)$  is an identity element in  $G$  with respect to  $' + '$

**Existence of Inverse:**

For each  $(a + \sqrt{5}b) \in G$  there exist  $(-a - \sqrt{5}b) \in G$

$$(a + \sqrt{5}b) + [(-a) + \sqrt{5}(-b)] = [a + (-a)] + \sqrt{5}[(-b) + b] = 0 + \sqrt{5}0 = \text{Identity}$$

$\therefore (-a) + \sqrt{5}(-b)$  is the inverse of  $a + \sqrt{5}b$

Hence  $G$  is a group with respect to  $' + '$ .

$$(iii) \begin{vmatrix} 4 & 4 & 4 & m \\ 4 & 4 & m & 4 \\ 4 & m & 4 & 4 \\ m & 4 & 4 & 4 \end{vmatrix} = 0$$

$$\begin{vmatrix} 4 & 4 & 4 & m + 12 \\ 4 & 4 & m & m + 12 \\ 4 & m & 4 & m + 12 \\ m & 4 & 4 & m + 12 \end{vmatrix} = 0 \quad \text{Add } C_1, C_2, C_3 \text{ in } C_4$$

$$(m + 12) \begin{vmatrix} 4 & 4 & 4 & 1 \\ 4 & 4 & m & 1 \\ 4 & m & 4 & 1 \\ m & 4 & 4 & 1 \end{vmatrix} = 0 \quad \text{Take } (m + 12) \text{ common from } C_4$$

$$R_2 - R_1, R_3 - R_1, R_4 - R_1$$

$$(m + 12) \begin{vmatrix} 4 & 4 & 4 & 1 \\ 0 & 0 & m - 4 & 0 \\ 0 & m - 4 & 0 & 0 \\ m - 4 & 0 & 0 & 0 \end{vmatrix} = 0$$

$$(m + 12)[(m - 4)^3] = 0$$

(Product of the principle diagonal elements in a triangular matrix)

$$\text{Either } (m + 12) = 0 \text{ or } (m - 4)^3 = 0$$

$$m = -12, 4$$

$$(iv) \text{ Let } P(x) = x^3 - 8sx^2 - 4tx + 9$$

As  $-2$  and  $2$  are the roots of  $P(x)$

$$\text{So } P(-2) = 0 \text{ and } P(2) = 0$$

$$\therefore P(-2) = (-2)^3 - 8s(-2)^2 - 4t(-2) + 9 = 0$$

$$-32s + 8t + 1 = 0$$

$$32s - 8t = 1 \quad \rightarrow \text{eqn(1)}$$

$$\text{Now } P(2) = (2)^3 - 8s(2)^2 - 4t(2) + 9 = 0$$

$$-32s - 8t + 17 = 0$$

$$32s + 8t = 17 \quad \rightarrow \text{eqn(2)}$$

On adding Equations (1) & (2), we get  $s = \frac{9}{32}$

On subtracting Equations (1) from (2), we get  $t = 1$

(v) If  $\alpha, \beta$  are the roots of  $x^2 - 3x + 2 = 0$  so  $\alpha + \beta = 3$  &  $\alpha\beta = 2$

If roots are  $\left(1 + \frac{3}{\alpha}\right)$  and  $\left(1 + \frac{3}{\beta}\right)$ , then

$$\text{Sum of roots: } S = \left(1 + \frac{3}{\alpha}\right) + \left(1 + \frac{3}{\beta}\right) = 2 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) = 2 + 3\left(\frac{3}{2}\right) = \frac{13}{2}$$

$$\text{Product of roots: } P = \left(1 + \frac{3}{\alpha}\right)\left(1 + \frac{3}{\beta}\right) = 1 + 3\left(\frac{\alpha+\beta}{\alpha\beta}\right) + \frac{9}{\alpha\beta} = 1 + 3\left(\frac{3}{2}\right) + \frac{9}{2} = 10$$

Required Equation:  $x^2 - Sx + P = 0$

$$x^2 - \frac{13}{2}x + 10 = 0$$

$$2x^2 - 13x + 20 = 0$$

$$(vi) \frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{A}{(3+x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2} \quad \rightarrow \text{eqn(1)}$$

$$7 - 3x + x^2 = A(1-x)^2 + B(3+x)(1-x) + C(3+x) \quad \rightarrow \text{eqn(2)}$$

For A put  $3+x=0$  or  $x=-3$  in eqn(2)

$$7 - 3(-3) + (-3)^2 = A(1+3)^2 + 0 + 0 \quad \Rightarrow A = \frac{25}{16}$$

For C put  $1-x=0$  or  $x=1$  in eqn(2)

$$7 - 3(1) + (1)^2 = 0 + 0 + C(3+1) \quad \Rightarrow C = \frac{5}{4}$$

Simplifying eqn(2) as

$$7 - 3x - x^2 = A(1+x^2 - 2x) + B(-x^2 - 2x + 3) + C(3+x)$$

Equating the coefficients of  $x^2$

$$-1 = A - B \quad \Rightarrow -1 = \frac{25}{16} - B \quad \Rightarrow B = \frac{9}{16}$$

Substituting the values of A, B and C in eqn (1)

$$\frac{7-3x+x^2}{(3+x)(1-x)^2} = \frac{25}{16(3+x)} + \frac{9}{16(1-x)} + \frac{5}{4(1-x)^2}$$

(vii) In A.P  $a_1 = 74$  and  $d = -6$

$$(a) S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_n = \frac{n}{2}[2(74) + (n-1)(-6)]$$

$$S_n = n(77 - 3n)$$

$$(b) S_n = n(77 - 3n)$$

$$380 = n(77 - 3n) \quad \because S_n = 380$$

$$3n^2 - 77n + 380 = 0$$

$$n = 19 \text{ or } \frac{20}{3}$$

$$\text{Since } n = \frac{20}{3} \notin \mathbb{N} \quad \Rightarrow \quad n = 19$$

(viii) G.P:  $a_1, a_1r, a_1r^2, a_1r^3, \dots, a_1r^{n-1}, \dots$

Given that  $a_2 = a_1r = \frac{1}{4} \Rightarrow a_1 = \frac{1}{4r} \rightarrow \text{eqn}(1)$

G.S:  $a_1 + a_1r + a_1r^2 + a_1r^3 + \dots + a_1r^{n-1} + \dots$

Taking  $S_\infty = \frac{a_1}{1-r}$

$$2 = \frac{a_1}{1-r} \quad \because S_\infty = 2$$

$$2(1-r) = a_1$$

Using eqn(1)

$$2(1-r) = \frac{1}{4r}$$

$$8r^2 - 8r + 1 = 0$$

$$r = \frac{2 \pm \sqrt{2}}{4}$$

Substituting it in eqn(1)

$$a_1 = \frac{1}{2 \pm \sqrt{2}}$$

(ix) Total arrangements =  $\frac{7!}{3!2!1!1!} = 420$

Numbers less than 20000000 (when we put 0 at extreme left position) =  $\frac{6!}{3!2!1!1!} = 60$

Numbers greater than 20000000 =  $420 - 60 = 360$

(x) Sample space  $n(S) = 52$

Let  $A$  be the event that the selected card is a heart card  $\Rightarrow n(A) = 13$

Let  $B$  be the event that the selected card is a face card  $\Rightarrow n(B) = 12$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} \quad \text{and} \quad P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$$

$A \cap B = \{\text{king of heart, jack of heart, queen of heart}\} \Rightarrow n(A \cap B) = 3$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$$

(xi)  $(1 - kx)^n = 1 - 10x + 60x^2 + \dots$

Comparing with

$$(1 - kx)^n = 1 - nkx + \frac{n(n-1)}{2!} k^2 x^2 + \dots$$

$$\text{We get } nk = 10 \Rightarrow k = \frac{10}{n} \quad \rightarrow \text{eqn}(1)$$

$$\text{and } \frac{n(n-1)}{2!} k^2 = 60 \Rightarrow \frac{n(n-1)}{2!} \left(\frac{10}{n}\right)^2 = 60 \quad \text{using eqn}(1)$$

$$\frac{5(n-1)}{n} = 6 \Rightarrow n = -5$$

$$k = \frac{10}{-5} = -2 \quad \text{Using eqn}(1)$$

$$(xii) \quad \frac{24 \cos \theta - 5 \sin^2 \theta}{\cos^2 \theta + 5 \cos \theta} = 5 - \sec \theta$$

$$L.H.S = \frac{24 \cos \theta - 5 \sin^2 \theta}{\cos^2 \theta + 5 \cos \theta}$$

$$L.H.S = \frac{24 \cos \theta - 5(1 - \cos^2 \theta)}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos^2 \theta + 24 \cos \theta - 5}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos^2 \theta + 25 \cos \theta - \cos \theta - 5}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{(5 \cos \theta - 1)(\cos \theta + 5)}{\cos \theta (\cos \theta + 5)}$$

$$L.H.S = \frac{5 \cos \theta}{\cos \theta} - \frac{1}{\cos \theta} = 5 - \sec \theta$$

$$(xiii) \quad \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$$

$$L.H.S = \frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$$

$$L.H.S = \frac{1 - (1 - 2 \sin^2 x) + 2 \sin x \cos x}{1 + (1 - 2 \sin^2 x) + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin^2 x + 2 \sin x \cos x}{2(1 - \sin^2 x) + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin x (\sin x + \cos x)}{2 \cos^2 x + 2 \sin x \cos x}$$

$$L.H.S = \frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\cos x + \sin x)}$$

$$L.H.S = \tan x$$

$$L.H.S = R.H.S$$

$$(xiv) \text{ Let } a_1 = a + d, b_1 = a + 2d, c_1 = a + 3d$$

$$\cos \gamma = \frac{a_1^2 + b_1^2 - c_1^2}{2 a_1 b_1}$$

$$\cos \gamma = \frac{(a + d)^2 + (a + 2d)^2 - (a + 3d)^2}{2 (a + d)(a + 2d)} \quad \text{substituting the values}$$

$$\cos \gamma = \frac{a^2 + d^2 + 2ad + a^2 + 4d^2 + 4ad - a^2 - 9d^2 - 6ad}{2 (a + d)(a + 2d)}$$

$$\cos \gamma = \frac{a^2 - 4d^2}{2 (a + d)(a + 2d)} = \frac{(a - 2d)(a + 2d)}{2(a + d)(a + 2d)} = \frac{a - 2d}{2(a + d)} = \frac{a}{2(a + d)} - \frac{d}{(a + d)}$$

$$\begin{aligned}
 \text{(xv)} \quad & \tan^{-1}\left(\frac{1+\frac{x}{2}}{1-\frac{x}{2}}\right) + \tan^{-1}\left(\frac{x}{2}\right) \\
 &= \tan^{-1}\left(\frac{2+x}{2-x}\right) + \tan^{-1}\left(\frac{x}{2}\right) \\
 &= \tan^{-1}\left(\frac{\frac{2+x}{2-x} + \frac{x}{2}}{1 - \left(\frac{2+x}{2-x}\right)\left(\frac{x}{2}\right)}\right) \quad \because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right) \\
 &= \tan^{-1}\left(\frac{2(2+x) + x(2-x)}{2(2-x) - x(2+x)}\right) \\
 &= \tan^{-1}\left(\frac{4+4x-x^2}{4-4x-x^2}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xvi)} \quad & 2 \cos 2\theta = -3 - 4\cos\theta \\
 & 2(2 \cos^2 \theta - 1) + 3 + 4\cos\theta = 0 \\
 & 4 \cos^2 \theta + 4\cos\theta + 1 = 0 \\
 & (2\cos\theta + 1)^2 = 0 \\
 & 2\cos\theta + 1 = 0 \\
 & \cos\theta = -\frac{1}{2}
 \end{aligned}$$

Here  $\cos\theta < 0 \Rightarrow \theta$  lies in Quadrants II & III with reference angle  $\frac{\pi}{3}$

$$\text{In Quad-II: } \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\text{In Quad-III: } \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\text{Solution Set} = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

### SECTION-C

$$\text{Q3. } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Using Row Operations

$$\begin{aligned}
 [A : I] &= \begin{bmatrix} 1 & 2 & 3 & : & 1 & 0 & 0 \\ 2 & 5 & 3 & : & 0 & 1 & 1 \\ 1 & 0 & 8 & : & 0 & 0 & 1 \end{bmatrix} \\
 R \sim \begin{bmatrix} 1 & 2 & 3 & : & 1 & 0 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & -2 & 5 & : & -1 & 0 & 1 \end{bmatrix} & \quad R_2 - 2R_1 ; R_3 - R_1 \\
 R \sim \begin{bmatrix} 1 & 0 & 9 & : & 5 & -2 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & 0 & -1 & : & -5 & 2 & 1 \end{bmatrix} & \quad R_1 - 2R_2 ; R_3 + 2R_2 \\
 R \sim \begin{bmatrix} 1 & 0 & 9 & : & 5 & -2 & 0 \\ 0 & 1 & -3 & : & -2 & 1 & 0 \\ 0 & 0 & 1 & : & 5 & -2 & -1 \end{bmatrix} & \quad (-1)R_3 \\
 R \sim \begin{bmatrix} 1 & 0 & 0 & : & -40 & 16 & 9 \\ 0 & 1 & 0 & : & 13 & -5 & -3 \\ 0 & 0 & 1 & : & 5 & -2 & -1 \end{bmatrix} & \quad R_1 - 9R_3 ; R_2 + 3R_3
 \end{aligned}$$



$$= [I : A^{-1}]$$

$$\text{Hence } A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Q4. (a) Number of men = 6, Number of women = 4

- (i) Number of ways that a 5 member Committee including exactly 3 men =  $\binom{6}{3} \times \binom{4}{2} = 120$   
 (ii) Number of ways that a 5 member Committee including at least 2 women  
 =  $\binom{6}{3} \times \binom{4}{2} + \binom{6}{2} \times \binom{4}{3} + \binom{6}{1} \times \binom{4}{4} = 120 + 60 + 6 = 166$   
 (b)  $S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

$$\Rightarrow n(S) = 12$$

Since A be the event that a Tail appear in tossing a coin.

$$A = \{(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\} \Rightarrow n(A) = 6$$

Since B be the event that 4 dots appear in rolling a dice.

$$B = \{(H, 4), (T, 4)\} \Rightarrow n(B) = 2$$

$$A \cap B = \{(T, 4)\} \Rightarrow n(A \cap B) = 1$$

$$P(A) = \frac{6}{12} = \frac{1}{2}, \quad P(B) = \frac{2}{12} = \frac{1}{6}, \quad P(A \cap B) = \frac{1}{12}$$

For Independent Events A and B

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{1}{12} = \left(\frac{1}{2}\right) \left(\frac{1}{6}\right)$$

$$\frac{1}{12} = \frac{1}{12}$$

Hence A and B are the independent events.

Q5.  $\left(\frac{3y^2}{2} - \frac{1}{3y}\right)^6$

$$T_{r+1} = \binom{n}{r} (a)^{n-r} (b)^r \quad \text{where } a = \frac{3y^2}{2}, b = -\frac{1}{3y}, n = 6$$

$$T_{r+1} = \binom{6}{r} \left(\frac{3y^2}{2}\right)^{6-r} \left(-\frac{1}{3y}\right)^r$$

$$T_{r+1} = \binom{6}{r} \frac{(3)^{6-r} y^{2(6-r)} (-1)^r}{(2)^{6-r} (3)^r (y)^r}$$

$$T_{r+1} = \binom{6}{r} \frac{(-1)^r (3)^{6-2r} (y)^{12-3r}}{(2)^{6-r}}$$

- (i) For the term involving  $y^3$  put  $12 - 3r = 3 \Rightarrow r = 3$

$$T_{3+1} = \binom{6}{3} \frac{(-1)^3 (3)^{6-6} (y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2} y^3$$

- (ii) For the term independent of  $y$  put  $12 - 3r = 0 \Rightarrow r = 4$

$$T_{4+1} = \binom{6}{4} \frac{(-1)^4 (3)^{6-8} (y)^{12-12}}{(2)^{6-4}} = \frac{5}{12}$$

(iii) Middle term =  $\left(\frac{n+2}{2}\right)th$  term

Middle term =  $\left(\frac{6+2}{2}\right)th = 4th$  term

$$T_{r+1} = \binom{6}{r} \frac{(-1)^r (3)^{6-2r} (y)^{12-3r}}{(2)^{6-r}}$$

$$T_{3+1} = \binom{6}{3} \frac{(-1)^3 (3)^{6-6} (y)^{12-9}}{(2)^{6-3}} = -\frac{5}{2}y^3$$

Q6. Statement: For any two angles  $\alpha$  and  $\beta$  (real numbers) then,

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Proof:

Consider four points A, B, C and D on a unit circle with center at O

Such that  $Arc \widehat{AB} = Arc \widehat{CD}$  and  $Chord \overline{AB} = Chord \overline{CD}$

Let  $\angle AOD = \alpha$  and  $\angle AOC = \beta$

Then  $\angle AOB = \alpha - \beta$ ,  $\angle COD = \alpha - \beta$

Here  $\alpha, \beta$  and  $\alpha - \beta$  are in standard position.

Co-ordinates of point A are (1,0)

Co-ordinates of point B are  $(\cos(\alpha - \beta), \sin(\alpha - \beta))$

Co-ordinates of point C are  $(\cos \beta, \sin \beta)$

Co-ordinate of point D are  $(\cos \alpha, \sin \alpha)$

Since  $|AB| = |CD| \quad \therefore Chord \overline{AB} = Chord \overline{CD}$

$$|AB|^2 = |CD|^2$$

$$[\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) - 0]^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$$

$$\cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta) = \cos^2 \alpha + \cos^2 \beta - 2\cos \alpha \cos \beta$$

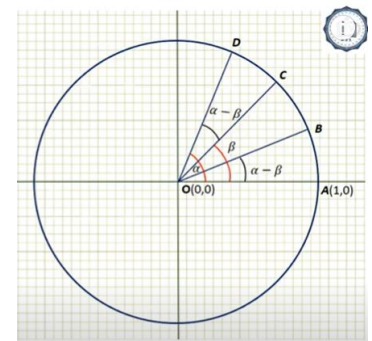
$$+ \sin^2 \alpha + \sin^2 \beta - 2\sin \alpha \sin \beta$$

$$[\cos^2(\alpha - \beta) + \sin^2(\alpha - \beta)] - 2\cos(\alpha - \beta) + 1 = (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta)$$

$$-2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$1 - 2\cos(\alpha - \beta) + 1 = 1 + 1 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



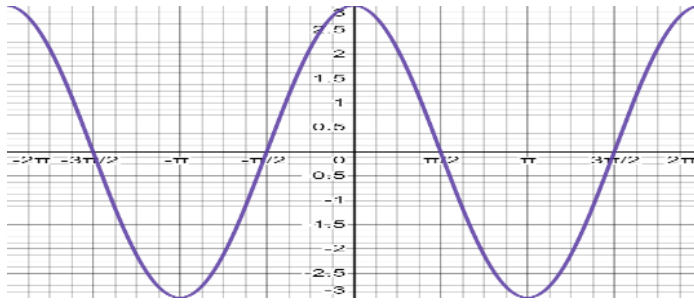
Q7.  $y = 3 \cos x$  for  $-2\pi \leq x \leq 2\pi$

(a)  $3 \cos x = 3 \cos(x + 2\pi) \quad \therefore$  Period of cosine function is  $2\pi$

Hence  $2\pi$  is the period of  $3 \cos x$

(b) Table of values

$x$	$0^\circ$	$\pm 30^\circ$	$\pm 60^\circ$	$\pm 90^\circ$	$\pm 120^\circ$	$\pm 150^\circ$	$\pm 180^\circ$
$y$	3	2.6	1.5	0	-1.5	-2.6	-3
$x$	$\pm 210^\circ$	$\pm 240^\circ$	$\pm 270^\circ$	$\pm 300^\circ$	$\pm 330^\circ$	$\pm 360^\circ$	$\pm 210^\circ$
$y$	-2.6	-1.5	0	1.5	2.6	3	-2.6



Q8.  $\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\left(\frac{117}{125}\right)$

Proof: L.H.S =  $\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right)$

L.H.S =  $\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right)$   $\because 2 \tan^{-1} A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

L.H.S =  $\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right)$   $\rightarrow \text{eqn(1)}$

Let  $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$  iff  $\cos \alpha = \frac{3}{5} > 0$  in  $[0, \pi] \Rightarrow \alpha \in \text{Quad - I}$

i.e.  $\sin \alpha = +\sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

Let  $\beta = \tan^{-1}\left(\frac{24}{7}\right)$  iff  $\tan \beta = \frac{24}{7} > 0$  in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \beta \in \text{Quad - I}$

$\sec \beta = +\sqrt{1 + \tan^2 \beta} = \sqrt{1 + \frac{576}{49}} = \frac{25}{7}$

$\sec \beta = \frac{25}{7} \Rightarrow \cos \beta = \frac{7}{25}$

$\sin \beta = +\sqrt{1 - \cos^2 \beta} = \sqrt{1 - \frac{49}{625}} = \frac{24}{25}$

$\because \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$\cos(\alpha - \beta) = \left(\frac{3}{5}\right)\left(\frac{7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right)$

$\cos(\alpha - \beta) = \frac{117}{125}$

$\alpha - \beta = \cos^{-1} \frac{117}{125}$

$\cos^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{24}{7}\right) = \cos^{-1} \frac{117}{125}$

$\cos^{-1}\left(\frac{3}{5}\right) - 2 \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1} \frac{117}{125}$