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Answer Sheet No. _____

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MATHEMATICS HSSC-I (2nd Set)

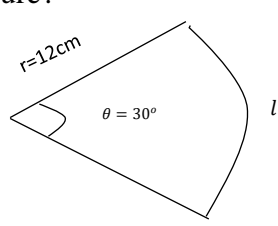
SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 Fill the relevant bubble for each part. All parts carry one mark.

- (1) If $\frac{2}{1-i} - x = 0$, then value of x is:
- A. $-1 - i$ B. $-1 + i$
 C. $1 - i$ D. $1 + i$
- (2) If A and B are two sets and $A \cap B = \phi$, then $n(A \cup B)$ is:
- A. $n(A) + n(B)$ B. $n(A)$
 C. $n(B)$ D. $n(A) + n(B) - n(A \cap B)$
- (3) If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then what is the order of matrix X ?
- A. 2×2 B. 2×3
 C. 3×2 D. 3×3
- (4) If $x^3 + 3x^2 - 6x + 2$ is divided by $x + 2$, then the remainder is:
- A. -18 B. 9
 C. -9 D. 18
- (5) If α, β are the roots of the equation $3x^2 - 2x - 9 = 0$, then $(\alpha + 1)(\beta + 1)$ is:
- A. $-\frac{2}{3}$ B. $\frac{2}{3}$
 C. $-\frac{1}{3}$ D. $\frac{1}{3}$
- (6) For how many values of x , the expression $x^2 - x - 2 = (x + 1)(x - 2)$ holds?
- A. For no value of x
 B. For only one value of x
 C. For only two values of x
 D. For all values of x

- (7) The series $1 + \frac{x}{2} + \frac{x^2}{2} + \dots$ is convergent if:
- A. $x \in R$ B. $x \in [-2, 2]$
 C. $x \in (-2, 2)$ D. $x \in Z$
- (8) Which of the following series represents $\sum_{n=1} 6(3)^{n-1}$?
- A. $6 + 9 + 12 + \dots$ B. $6 + 18 + 54 + \dots$
 C. $3 + 9 + 27 + \dots$ D. $6 + 12 + 18 + \dots$
- (9) The probability of getting a total of 10 in a single throw of two dice is:
- A. $\frac{1}{9}$ B. $\frac{1}{12}$
 C. $\frac{1}{6}$ D. $\frac{5}{36}$
- (10) In how many ways can we choose a committee of 5 from 8 persons?
- A. 56 B. 336
 C. 6720 D. 6
- (11) The middle term in the expansion of $(a + b)^6$ is:
- A. T_3 B. T_4
 C. T_5 D. T_6
- (12) The expansion of $(1 - 2x)^{\frac{1}{3}}$ is valid if
- A. $|x| > \frac{1}{2}$ B. $|x| > 1$
 C. $|x| < \frac{1}{2}$ D. $|x| < 2$
- (13) What is the value of l in the adjoining figure?
- A. π B. 2π
 C. 3π D. 4π
- 
- (14) $\sin 294^\circ =$ _____.
- A. $\sin 24^\circ$ B. $\cos 24^\circ$
 C. $-\sin 24^\circ$ D. $-\cos 24^\circ$
- (15) Which one of the following is equal to $\cos(\alpha + \beta)$ if $\alpha + \beta + \gamma = 180^\circ$?
- A. $\sin \gamma$ B. $\cos \gamma$
 C. $-\cos \gamma$ D. $-\sin \gamma$
- (16) At what angle, the graph of $y = \cos 2x$ crosses x -axis?
- A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$
 C. π D. 0
- (17) If $a = 2$, $b = 3$ and $\gamma = 30^\circ$, then triangular area is:
- A. 1.5 B. 0.8
 C. 2.6 D. 2.1

Model Question Paper HSSC-I

Mathematics

(2nd Set)SOLUTION

Section – A

1	C	2	B	3	C	4	B	5	A	6	B	7	D	8	B	9	A	10	A
11	B	12	D	13	C	14	B	15	B	16	D	17	B	18	A	19	B	20	D

SECTION-B

Q2.

(i) If $z = \sqrt{2} - i$ then $\bar{z} = \sqrt{2} + i$

a) $z^2 + \bar{z}^2 = (\sqrt{2} - i)^2 + (\sqrt{2} + i)^2$

$$z^2 + \bar{z}^2 = 2 - 2\sqrt{2}i + i^2 + 2 + 2\sqrt{2}i + i^2$$

$$z^2 + \bar{z}^2 = 2 + (-1) + 2 + (-1) = 0 \in \mathcal{R}$$

b) $(z - \bar{z})^2 = (\sqrt{2} - i - \sqrt{2} - i)^2$

$$(z - \bar{z})^2 = (-2i)^2$$

$$(z - \bar{z})^2 = 4i^2 = -4 \in \mathcal{R}$$

(ii) $p \rightarrow q = \sim (p \wedge \sim q)$

p	q	$\sim q$	$p \wedge \sim q$	$\sim (p \wedge \sim q)$	$p \rightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

From column 4 and column 6: $p \rightarrow q = \sim (p \wedge \sim q)$

$$(iii) \quad A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$$

$$a) \quad A_{11} = \begin{vmatrix} -2 & 2 \\ 2 & -3 \end{vmatrix} = 6 - 4 = 2$$

$$A_{21} = \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} = -6 + 1 = -5$$

$$A_{31} = \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} = 4 + 2 = 6$$

$$b) \quad |A| = A_{11} - A_{21} + A_{31} = 2 + 5 + 6 = 13$$

$$(iv) \quad y + 2 = 11x \quad \Rightarrow \quad y = 11x - 2 \quad \dots\dots\dots \text{eqn-I}$$

$$y = 25x^2 - 9x + 2$$

Using eqn-I

$$11x - 2 = 25x^2 - 9x + 2$$

$$25x^2 - 20x + 4 = 0$$

$$(5x - 2)^2 = 0$$

$$x = \frac{2}{5}$$

Substitute value of x in eq-I

$$y = 11 \left(\frac{2}{5} \right) - 2 = \frac{12}{5}$$

$$\text{Solution Set} = \left\{ \left(\frac{2}{5}, \frac{12}{5} \right) \right\}$$

$$(v) \quad (x - p)(x - q) + (x - q)(x - r) + (x - r)(x - p) = 0$$

$$x^2 - (p + q)x + pq + x^2 - (q + r)x + qr + x^2 - (r + p)x + rp = 0$$

$$3x^2 - 2(p + q + r)x + (pq + qr + rp) = 0$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= [-2(p + q + r)]^2 - 4(3)(pq + qr + rp)$$

$$= 4(p^2 + q^2 + r^2 + 2pq + 2qr + 2rp - 3pq - 3qr - 3rp)$$

$$= 4(p^2 + q^2 + r^2 - pq - qr - rp)$$

$$= 2(2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp)$$

$$= 2[p^2 + p^2 + q^2 + q^2 + r^2 + r^2 - 2pq - 2qr - 2rp]$$

$$= 2[p^2 + q^2 - 2pq + q^2 + r^2 - 2qr + r^2 + p^2 - 2rp]$$

$$= 2[(p - q)^2 + (q - r)^2 + (r - p)^2] > 0$$

i.e. roots of the given equation are real.

For equal roots

$$\text{Discriminant} = 0$$

$$2[(p - q)^2 + (q - r)^2 + (r - p)^2] = 0$$

$$(p - q)^2 + (q - r)^2 + (r - p)^2 = 0$$

This is only possible if

$$(p - q)^2 = 0 \Rightarrow p = q$$

$$(q - r)^2 = 0 \Rightarrow q = r$$

$$(r - p)^2 = 0 \Rightarrow r = p$$

Hence $p = q = r$

$$(vi) \quad \frac{2x-3}{(x^2-x+1)(3x-2)} = \frac{Ax+B}{(x^2-x+1)} + \frac{C}{(3x-2)} \text{-----eqn-I}$$

$$2x - 3 = (Ax + B)(3x - 2) + C(x^2 - x + 1) \text{-----eqn-II}$$

For C put $3x - 2 = 0$ or $x = \frac{2}{3}$ in eqn-I

$$2\left(\frac{2}{3}\right) - 3 = 0 + C\left(\frac{4}{9} - \frac{2}{3} + 1\right)$$

$$\frac{4}{3} - 3 = C\left(-\frac{2}{9} + 1\right) \Rightarrow C = -\frac{15}{7}$$

Simplifying eqn-II as

$$2x - 3 = A(3x^2 - 2x) + B(3x - 2) + C(x^2 - x + 1)$$

Equating the coefficients of x^2 , constant terms

$$0 = 3A + C \qquad -3 = -2B + C$$

$$0 = 3A - \frac{15}{7} \qquad -3 = -2B - \frac{15}{7}$$

$$A = -\frac{5}{7} \qquad B = \frac{3}{7}$$

Substituting the values in eqn - I

$$\frac{2x-3}{(x^2-x+1)(3x-2)} = \frac{-\frac{5}{7}x + \frac{3}{7}}{(x^2-x+1)} + \frac{-\frac{15}{7}}{(3x-2)}$$

$$\frac{2x-3}{(x^2-x+1)(3x-2)} = \frac{-5x+3}{7(x^2-x+1)} - \frac{15}{7(3x-2)}$$

(vii) A.P: b, c, p, q, r

Consider the following

$$a_1 + a_5 = b + r$$

$$a_2 + a_4 = c + q$$

$$2a_3 = 2p$$

$$a + (a + 4d) = b + r$$

$$a + d + a + 3d = c + q$$

$$2(a + 2d) = 2p$$

$$2a + 4d = b + r$$

$$2a + 4d = c + q$$

$$2a + 4d = 2p$$

From above $b + r = c + q = 2p$

(viii) In HP: $a_p = q$; $a_q = p$; $a_{pq} = ?$

$$\text{In AP: } a_p = \frac{1}{q} ; a_q = \frac{1}{p}$$

$$\therefore a_n = a_1 + (n - 1)d$$

$$a_p = a_1 + (p - 1)d$$

$$a_q = a_1 + (q - 1)d$$

$$\frac{1}{q} = a_1 + pd - d \text{ -----eqn-I}$$

$$\frac{1}{p} = a_1 + qd - d$$

$$\frac{1}{q} - \frac{1}{p} = (a_1 + pd - d) - (a_1 + qd - d)d$$

$$\frac{1}{q} - \frac{1}{p} = (p - q)d$$

$$\frac{p - q}{pq} = (p - q)d$$

$$d = \frac{1}{pq}$$

Put the value of d in eqn-I giving $a_1 = \frac{1}{pq}$

$$a_{pq} = a_1 + (pq - 1)d$$

$$a_{pq} = \frac{1}{pq} + (pq - 1) \frac{1}{pq}$$

$$a_{pq} = \frac{1}{pq} + 1 - \frac{1}{pq} = 1$$

$$a_{pq} = 1$$

Thus (pq) th term of H.P = 1

(ix) (a) when C and K are placed together

$$\text{Number of permutations} = 2! \times 5! = 2 \times 120 = 240$$

(b) Total number of permutation = $6! = 720$

when C and K are not placed together

Number of permutations

$$= \text{Total number of permutation} - \text{Number of permutation when C and K are together}$$

$$= 720 - 240 = 480$$

(x) Let, $a > b$ and $= b + h$, where h is so small that its square and higher power be neglected.

$$\begin{aligned}
 \text{L.H.S} &= \frac{b+2a}{a+2b} = \frac{b+2(b+h)}{(b+h)+2b} = \frac{3b+2h}{3b+h} = \frac{1+\frac{2h}{3b}}{1+\frac{h}{3b}} \\
 &= \left(1 + \frac{2h}{3b}\right) \left(1 + \frac{h}{3b}\right)^{-1} \\
 &= \left(1 + \frac{2h}{3b}\right) \left(1 + (-1)\frac{h}{3b} + \frac{(-1)(-2)}{2!} \left(\frac{h}{3b}\right)^2 + \dots\right) \\
 &\approx \left(1 + \frac{2h}{3b}\right) \left(1 - \frac{h}{3b}\right) \\
 &\approx 1 - \frac{h}{3b} + \frac{2h}{3b}
 \end{aligned}$$

$$\text{L.H.S} \approx 1 + \frac{h}{3b}$$

$$\text{R.H.S.} = \sqrt[3]{\frac{a}{b}} = \left(\frac{b+h}{b}\right)^{\frac{1}{3}} = \left(1 + \frac{h}{b}\right)^{\frac{1}{3}} \approx 1 + \frac{h}{3b}$$

Hence L.H.S = R.H.S

(xi) In the given right triangle where θ lies in the first quadrant

Let $x = 12$, $y = 5$, $r = 13$

$$\sec\theta = \frac{r}{x} = \frac{13}{12} \quad \operatorname{cosec}\theta = \frac{r}{y} = \frac{13}{5} \quad \tan\theta = \frac{y}{x} = \frac{5}{12} \quad \cot\theta = \frac{x}{y} = \frac{12}{5}$$

$$\text{a) } \sec^2\theta - \tan^2\theta = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2 = \frac{169}{144} - \frac{25}{144} = \frac{144}{144} = 1$$

$$\text{b) } \operatorname{cosec}^2\theta - \cot^2\theta = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 = \frac{169}{25} - \frac{144}{25} = \frac{25}{25} = 1$$

$$\text{(xii) } \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

$$\begin{aligned}
 \text{L.H.S} &= \tan(\alpha - \beta) \\
 &= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} \\
 &= \frac{\sin\alpha \cos\beta - \cos\alpha \sin\beta}{\cos\alpha \cos\beta - \sin\alpha \sin\beta}
 \end{aligned}$$

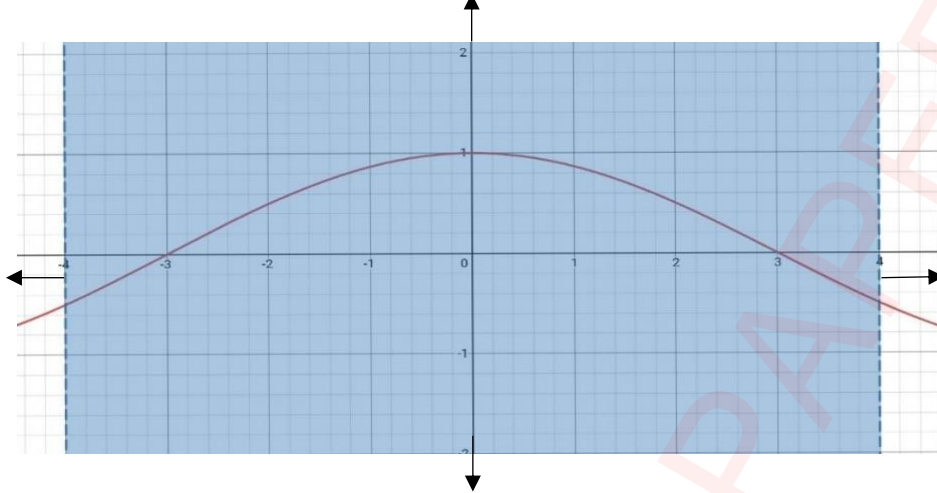
Dividing the numerator and denominator by $\cos\alpha \cos\beta$

$$\begin{aligned}
 &= \frac{\left(\frac{\sin\alpha \cos\beta - \cos\alpha \sin\beta}{\cos\alpha \cos\beta}\right)}{\left(\frac{\cos\alpha \cos\beta - \sin\alpha \sin\beta}{\cos\alpha \cos\beta}\right)} \\
 &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}
 \end{aligned}$$

= R.H.S.

(xiii) $y = \cos\left(\frac{\pi}{6} x\right) \quad -4 \leq x \leq 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	-0.5	0	0.5	0.87	1	0.87	0.5	0	0.5



(xiv) $L.H.S = \frac{\cos \alpha}{a} + \frac{\cos \beta}{b} + \frac{\cos \gamma}{c}$

By Law of cosines

$$\cos \alpha = \frac{b^2+c^2-a^2}{2bc} ; \cos \beta = \frac{a^2+c^2-b^2}{2ac} ; \cos \gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$\begin{aligned} L.H.S &= \frac{b^2+c^2-a^2}{2abc} + \frac{a^2+c^2-b^2}{2acb} + \frac{a^2+b^2-c^2}{2abc} \\ &= \frac{1}{2abc} [b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2] \\ &= \frac{a^2+b^2+c^2}{2abc} = \text{R.H.S} \end{aligned}$$

(xv) $L.H.S = 4 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$$= 2 \left[2 \tan^{-1}\left(\frac{1}{5}\right) \right] + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= 2 \left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\left(\frac{2}{5}\right)^2}\right) \right] + \tan^{-1}\left(\frac{1}{239}\right)$$

$$\because 2 \tan^{-1} A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$$

$$= 2 \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{239}\right)$$

$$= \tan^{-1}\left(\frac{2 \times \frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}\right) + \tan^{-1}\left(\frac{1}{239}\right)$$

$$\begin{aligned}
&= \tan^{-1} \left(\frac{120}{119} \right) + \tan^{-1} \left(\frac{1}{239} \right) \\
&= \tan^{-1} \left(\frac{\frac{120}{119} + \frac{1}{239}}{1 - \left(\frac{120}{119} \times \frac{1}{239} \right)} \right) \quad \because \tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right) \\
&= \tan^{-1} \left(\frac{28680-119}{28441+120} \right) = \tan^{-1} \left(\frac{28561}{28561} \right) \\
&= \tan^{-1}(1) \\
&= \frac{\pi}{4} = R.H.S
\end{aligned}$$

(xvi) $\sin x + \cos x = 1$

Taking square of both sides

$$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1$$

$$1 + 2\sin x \cos x = 1$$

$$2\sin x \cos x = 0$$

$$\sin 2x = 0$$

$$2x = n\pi \quad \text{or} \quad x = \frac{n\pi}{2} \quad \text{where } n \in \mathbb{Z}$$

$$\text{Solution set} = \left\{ \frac{n\pi}{2} \right\}, \quad n \in \mathbb{Z}$$

SECTION-C

Q3. $x + 7y - z = 10$

$$4x + 8y + z = 5$$

$$2x - 3y + 2z = -5$$

$$\begin{bmatrix} 1 & 7 & -1 & 10 \\ 4 & 8 & 1 & 5 \\ 2 & -3 & 2 & -5 \end{bmatrix}$$

$$R \sim \begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & -20 & 5 & -35 \\ 0 & -17 & 4 & -25 \end{bmatrix} \quad R_2 - 4R_1 ; \quad R_3 - 2R_1$$

$$R \sim \begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & -17 & 4 & -25 \end{bmatrix} \quad -\frac{1}{20}R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & -\frac{1}{4} & \frac{19}{4} \end{bmatrix} \quad R_1 - 7R_2 ; R_3 + 17R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 1 & -19 \end{bmatrix} \quad -4R_3$$

$$R \sim \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -19 \end{bmatrix} \quad \frac{1}{4}R_3 + R_2 ; -\frac{3}{4}R_3 + R_1$$

$$x = 12, y = -3, z = -19$$

Q4. $ax^2 + bx + c = 0$, ($a \neq 0$)

(i) Let α and 3α be the roots of the equation

$$\text{Sum of the roots: } \alpha + 3\alpha = -\frac{b}{a}$$

$$4\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{4a}$$

$$\text{Product of the roots: } (\alpha)(3\alpha) = \frac{c}{a}$$

$$3\alpha^2 = \frac{c}{a}$$

$$3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a} \quad \because \alpha = -\frac{b}{4a}$$

$$\Rightarrow 3b^2 = 16ca$$

(ii) Let α and α^2 be the roots of the equation

$$\text{Product of the roots: } (\alpha)(\alpha^2) = \frac{c}{a}$$

$$\alpha^3 = \frac{c}{a} \Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

$$\text{Sum of the roots: } \alpha + \alpha^2 = -\frac{b}{a}$$

$$\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left(\frac{c}{a}\right)^{\frac{2}{3}} = -\frac{b}{a} \quad \because \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

(iii) Let α and $-\alpha$ be the roots of the equation, then

$$\text{Product of the roots: } (\alpha)(-\alpha) = \frac{c}{a}$$

$$\alpha^2 = -\frac{c}{a}$$

$$\text{Sum of the roots: } \alpha + (-\alpha) = -\frac{b}{a}$$

$$0 = -\frac{b}{a}$$

$$b = 0$$

(iv) Let α and $\frac{1}{\alpha}$ be the roots of the equation.

$$\text{Sum of the roots: } \alpha + \frac{1}{\alpha} = -\frac{b}{a}$$

$$\text{Product of the roots: } (\alpha)\left(\frac{1}{\alpha}\right) = \frac{c}{a}$$

$$1 = \frac{c}{a}$$

$$c = a$$

Q5. Let $x = \left(2^{\frac{1}{4}}\right)\left(4^{\frac{1}{8}}\right)\left(8^{\frac{1}{16}}\right)\left(16^{\frac{1}{32}}\right) \dots \infty$

Taking log on both sides

$$\log x = \log 2^{\frac{1}{4}} + \log 4^{\frac{1}{8}} + \log 8^{\frac{1}{16}} + \log 16^{\frac{1}{32}} + \dots \infty$$

$$\log x = \log 2^{\frac{1}{4}} + \log (2^2)^{\frac{1}{8}} + \log (2^3)^{\frac{1}{16}} + \log (2^4)^{\frac{1}{32}} + \dots \infty$$

$$\log x = \frac{1}{4} \log 2 + \frac{2}{8} \log 2 + \frac{3}{16} \log 2 + \frac{4}{32} \log 2 + \dots \infty$$

$$\log x = \left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots \infty\right) \log 2 \quad \dots \dots \dots \quad \text{(i)}$$

$$\text{Let } S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots \infty \quad \dots \dots \dots \quad \text{(ii)}$$

Multiplying by $\left(\frac{1}{2}\right)$

$$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{4}{64} + \dots \infty \quad \dots \dots \dots \quad \text{(iii)}$$

eq. (ii) – eq. (iii)

$$\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \infty$$

$$\frac{1}{2}S = S_{\infty}$$

$$\text{Where } S_{\infty} = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \dots \infty$$

$$\text{Here } a_1 = \frac{1}{2^2} \text{ and } r = \frac{1}{2}$$

$$\therefore S_{\infty} = \frac{a_1}{1-r} = \frac{\frac{1}{2^2}}{\frac{1}{2}} = \frac{1}{2}$$

$$\text{Now } \frac{1}{2}S = \frac{1}{2}$$

$$S = 1$$

Put it in eq. (i)

$$\log x = (S)\log 2$$

$$\log x = (1)\log 2$$

$$\log x = \log 2$$

$$x = 2$$

Q6. $3^n + 2^{n-1} < 4^n$

Step 1: Let the given inequality is true for $n = 2$

$$3^2 + 2^1 < 4^2$$

$$9 + 2 < 16$$

$$11 < 16 \text{ (true)}$$

Step 2: Let the statement be true for $n = k \geq 2$

$$3^k + 2^{k-1} < 4^k \quad \dots\dots\dots \text{(i)}$$

Step 3: Multiplying both sides by 4

$$4(3^k + 2^{k-1}) < 4(4^k)$$

$$4 \cdot 3^k + 4 \cdot 2^{k-1} < 4 \cdot 4^k$$

$$(3 + 1) \cdot 3^k + (2 + 2) \cdot 2^{k-1} < 4^{k+1}$$

$$3 \cdot 3^k + 3^k + 2 \cdot 2^{k-1} + 2 \cdot 2^{k-1} < 4^{k+1}$$

$$3^{k+1} + 2^k + 3^k + 2^k < 4^{k+1}$$

$$3^{\overline{k+1}} + 2^{\overline{k+1}-1} < 4^{\overline{k+1}}$$

Which is true for $n = k + 1$.

The truth for $n = k$ implies its truth for $n = k + 1$.

Hence it is true for all positive integral values of n .

Q7.

(i) $\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta = 4\cos\theta \sin 6\theta \cos 2\theta$

$$\text{L.H.S.} = \sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta$$

Using $\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$$\text{L.H.S.} = 2 \sin\left(\frac{3\theta+5\theta}{2}\right) \cos\left(\frac{3\theta-5\theta}{2}\right) + 2 \sin\left(\frac{7\theta+9\theta}{2}\right) \cos\left(\frac{7\theta-9\theta}{2}\right)$$

$$\text{L.H.S.} = 2 \sin 4\theta \cos \theta + 2 \sin 8\theta \cos \theta$$

$$\text{L.H.S.} = 2\cos\theta(\sin 4\theta + \sin 8\theta)$$

$$\text{L.H.S.} = 2\cos\theta \left\{ 2 \sin\left(\frac{4\theta+8\theta}{2}\right) \cos\left(\frac{4\theta-8\theta}{2}\right) \right\}$$

$$\text{L.H.S.} = 4\cos\theta \sin 6\theta \cos 2\theta = \text{R.H.S.}$$

(ii) $\cos 5\theta + \cos\theta + 2\cos 3\theta = 4\cos 3\theta \cos^2\theta$

$$\text{L.H.S.} = \cos 5\theta + \cos\theta + 2\cos 3\theta$$

$$\text{Using } \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.} = 2 \cos\left(\frac{5\theta+\theta}{2}\right) \cos\left(\frac{5\theta-\theta}{2}\right) + 2\cos 3\theta$$

$$\text{L.H.S.} = 2 \cos 3\theta \cos 2\theta + 2 \cos 3\theta$$

$$\text{L.H.S.} = 2\cos 3\theta(\cos 2\theta + 1)$$

$$\text{L.H.S.} = 2\cos 3\theta(2 \cos^2 \theta)$$

$$\because \cos 2\theta + 1 = 2 \cos^2 \theta$$

$$\text{L.H.S.} = 4\cos 3\theta \cos^2 \theta = \text{R.H.S.}$$

Q8. Let height of the poster: $|DE| = 4 \text{ feet}$

Height of the man: $|AF| = 5 \text{ feet}$

Angle of Observation: $\angle D\hat{E}F = \theta = \theta_1 - \theta_2 = ?$

$$\text{In rt}\Delta EFC \quad \tan \theta_1 = \frac{7}{x}$$

$$\text{In rt}\Delta DFC \quad \tan \theta_2 = \frac{3}{x}$$

$$\tan(\theta_1 - \theta_2) = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$$

$$\tan(\theta) = \frac{\frac{7}{x} - \frac{3}{x}}{1 + \left(\frac{7}{x}\right)\left(\frac{3}{x}\right)} = \frac{\frac{4}{x}}{\frac{x^2 + 21}{x^2}} = \frac{4x}{x^2 + 21}$$

$$\theta = \tan^{-1}\left(\frac{4x}{x^2 + 21}\right)$$

